Designing Surprise Bags for Surplus Foods

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Joint work with
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Source: rednote

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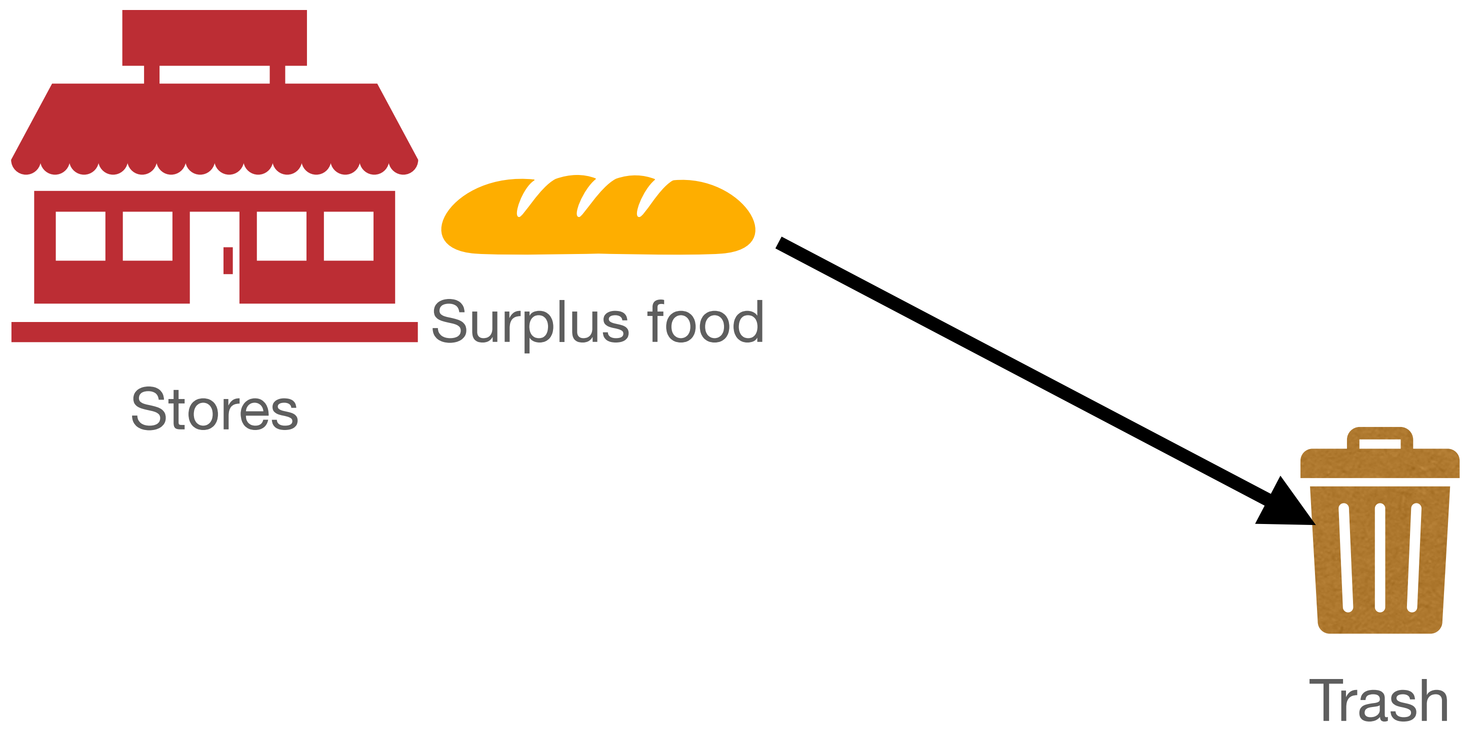

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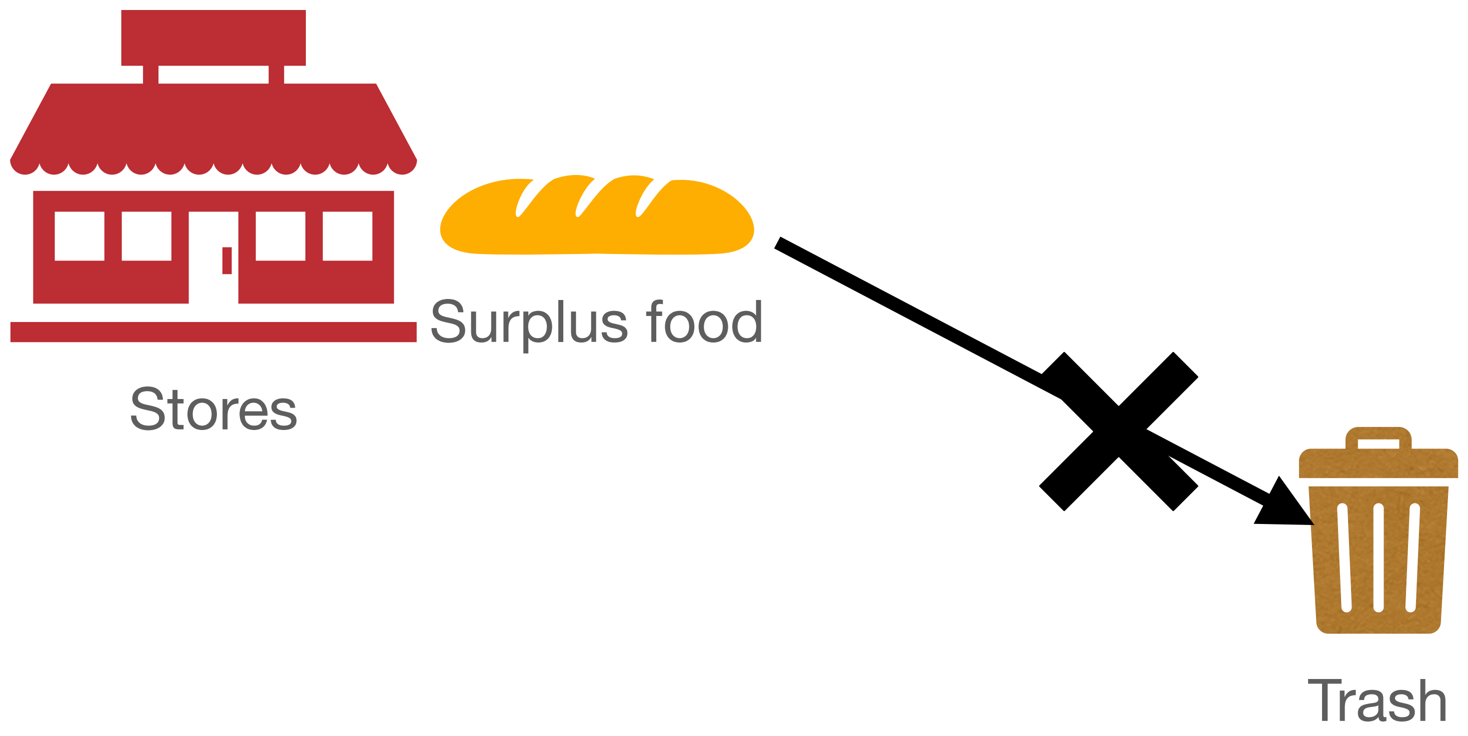

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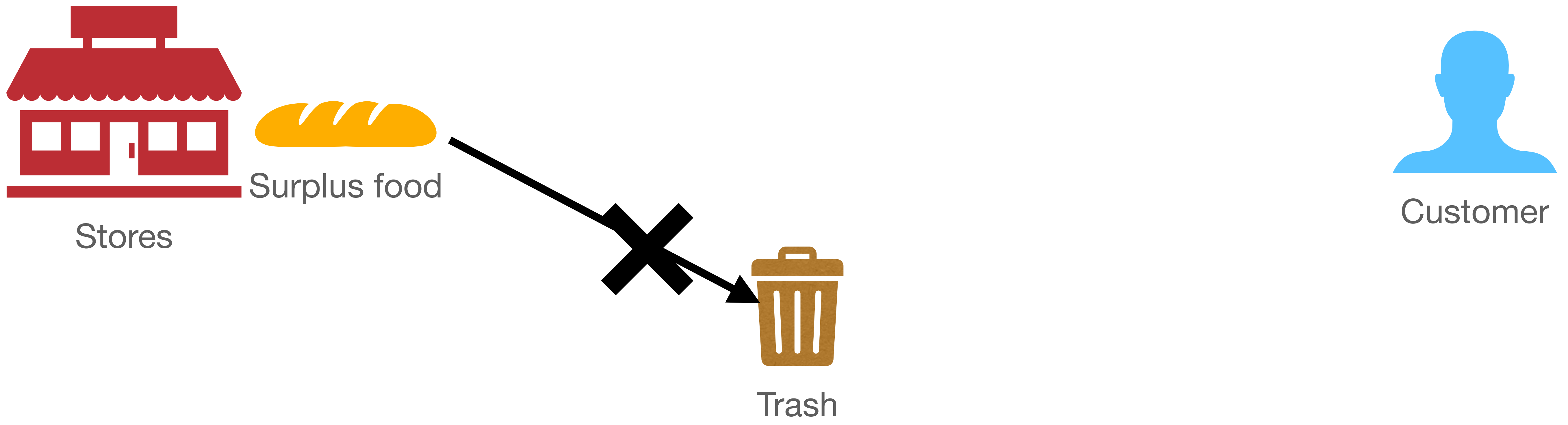

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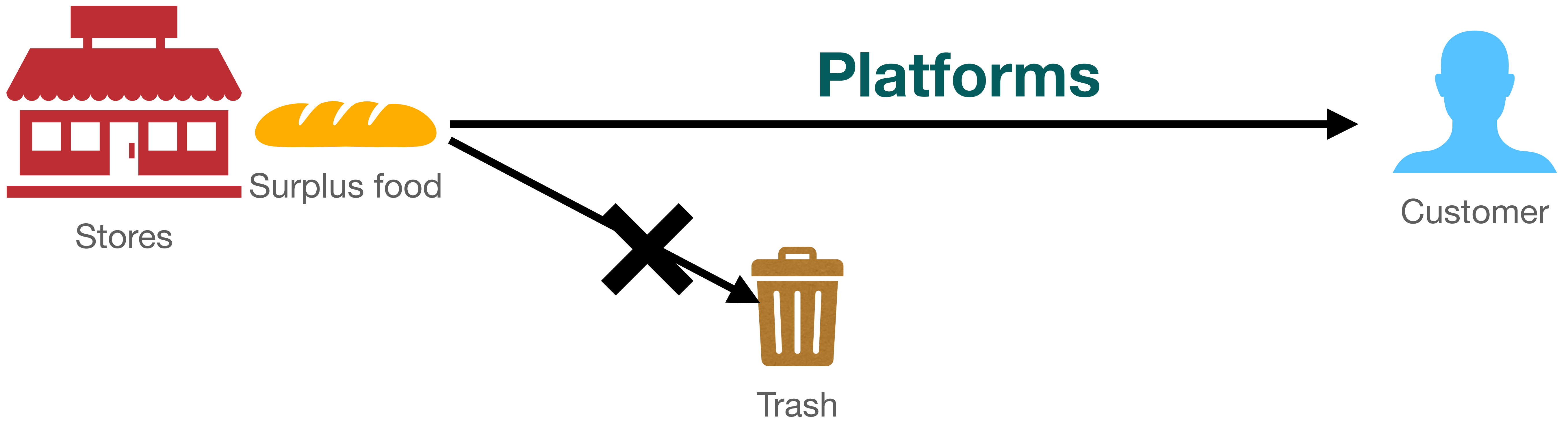

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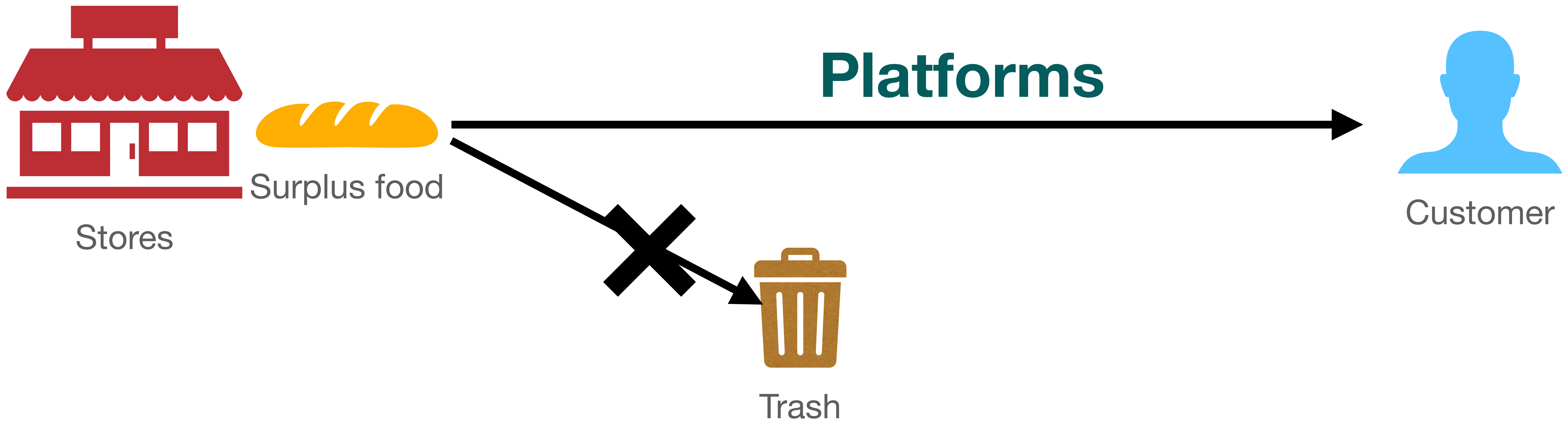

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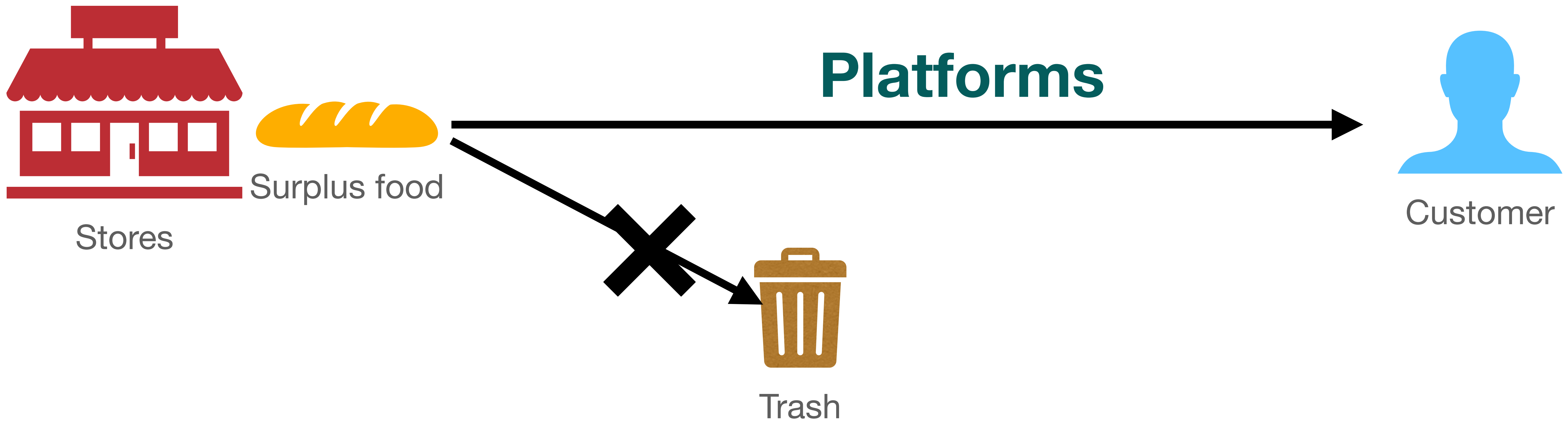

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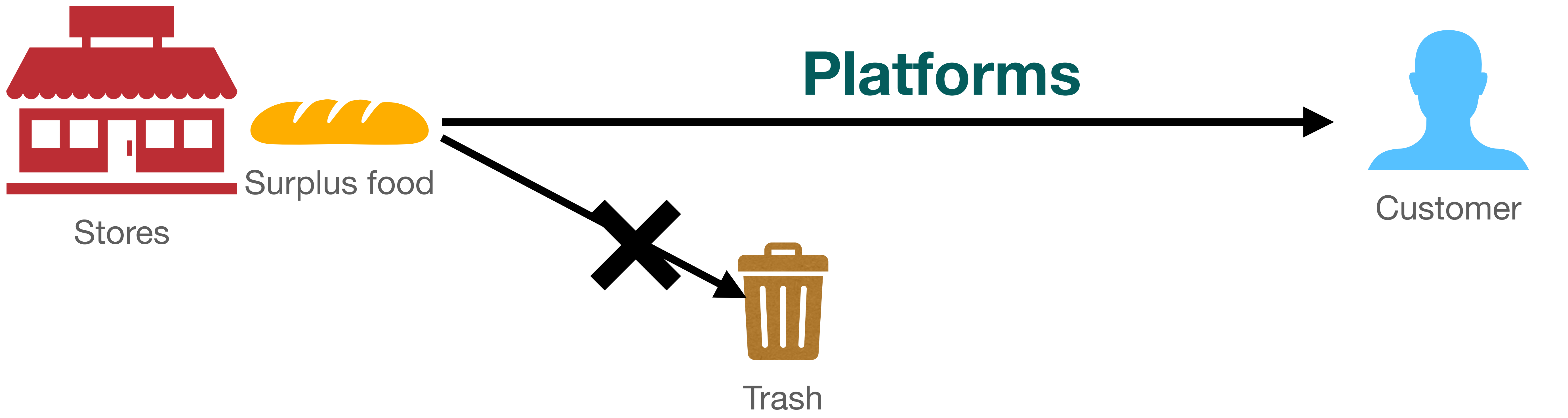

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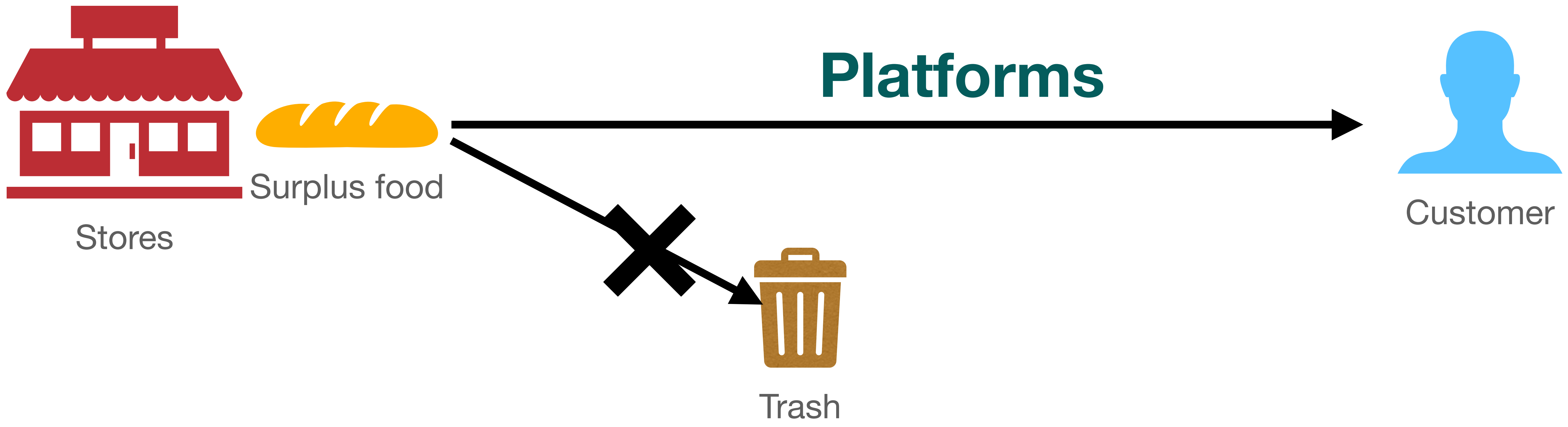

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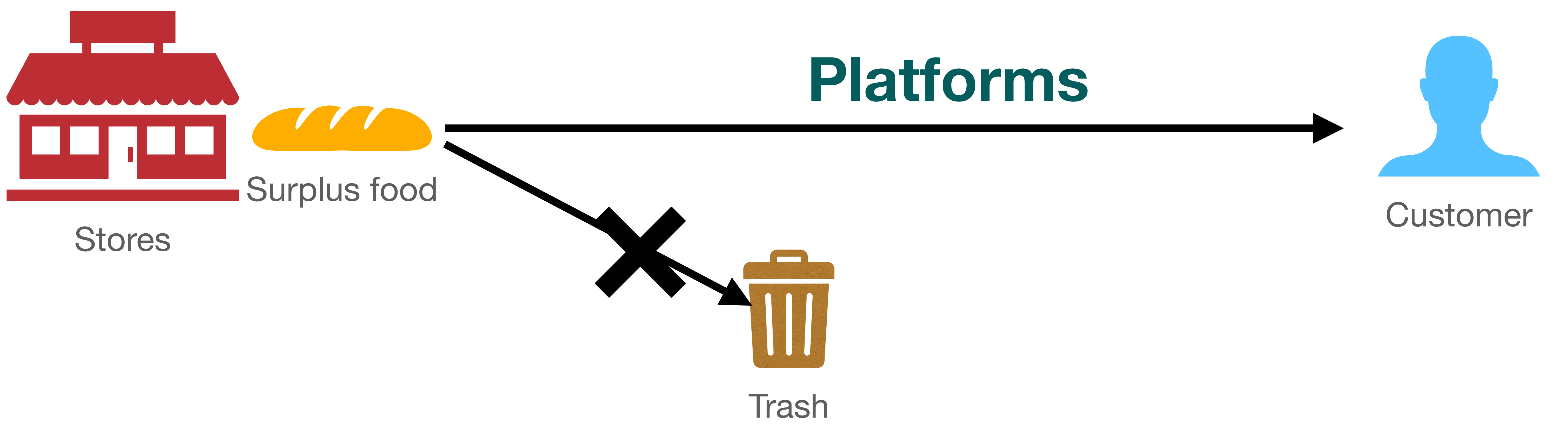

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Customer

Enjoy discounted food

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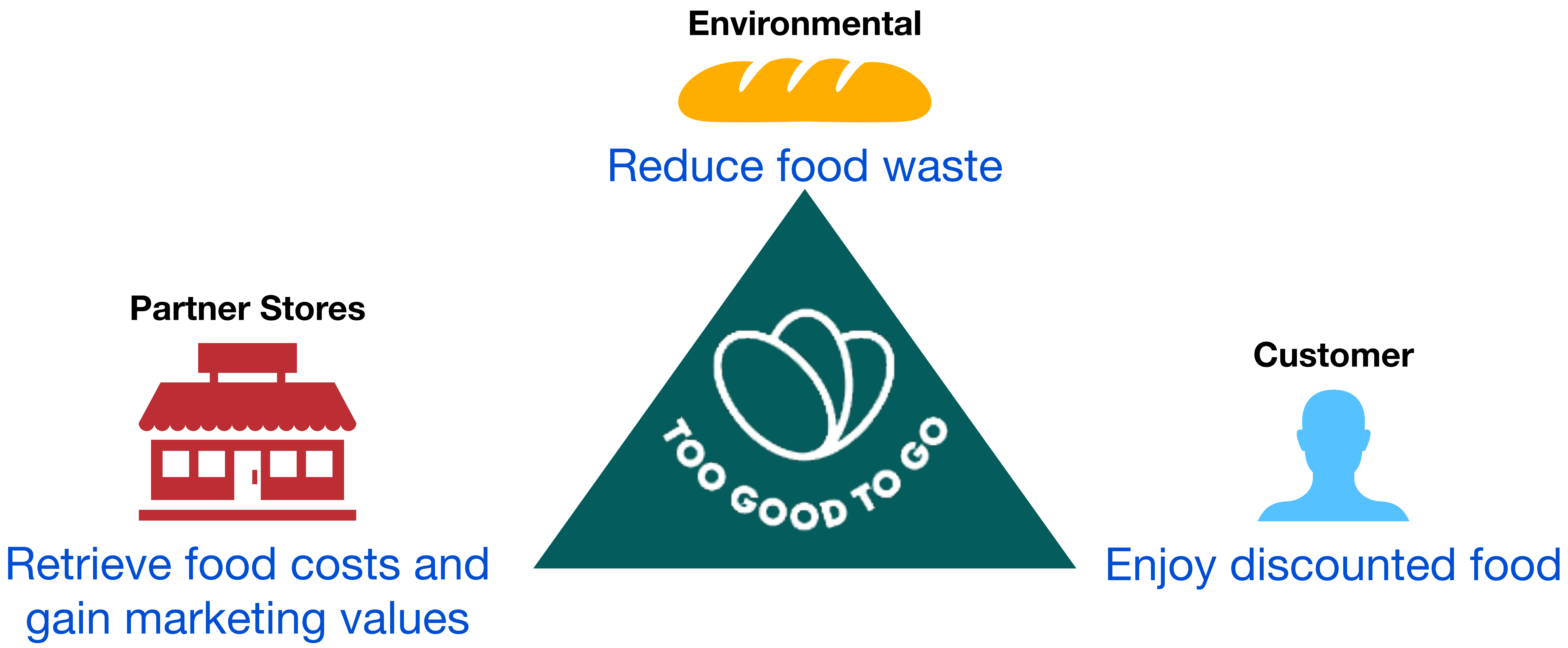
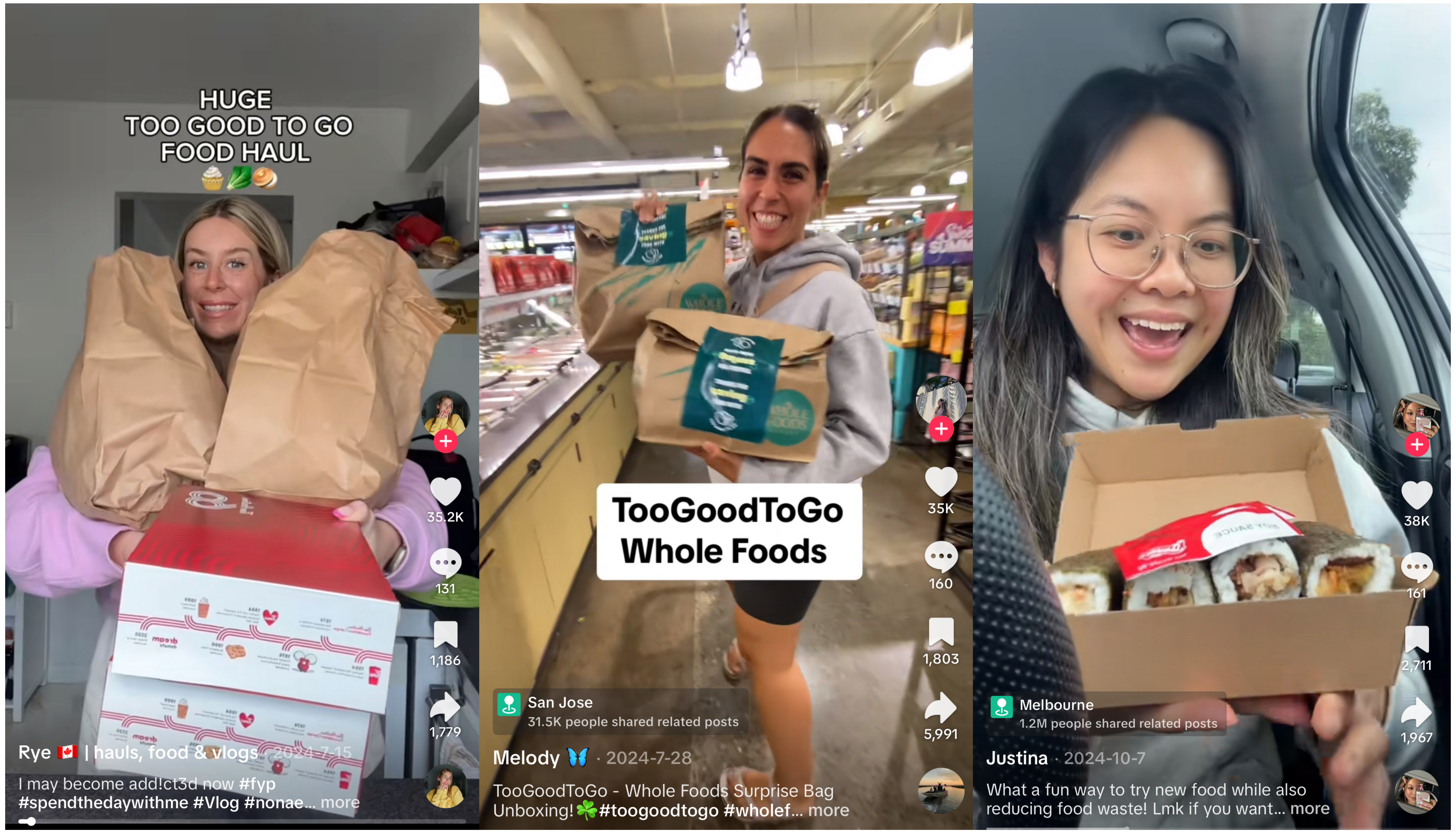


Surprise Bags

Viral social media influence



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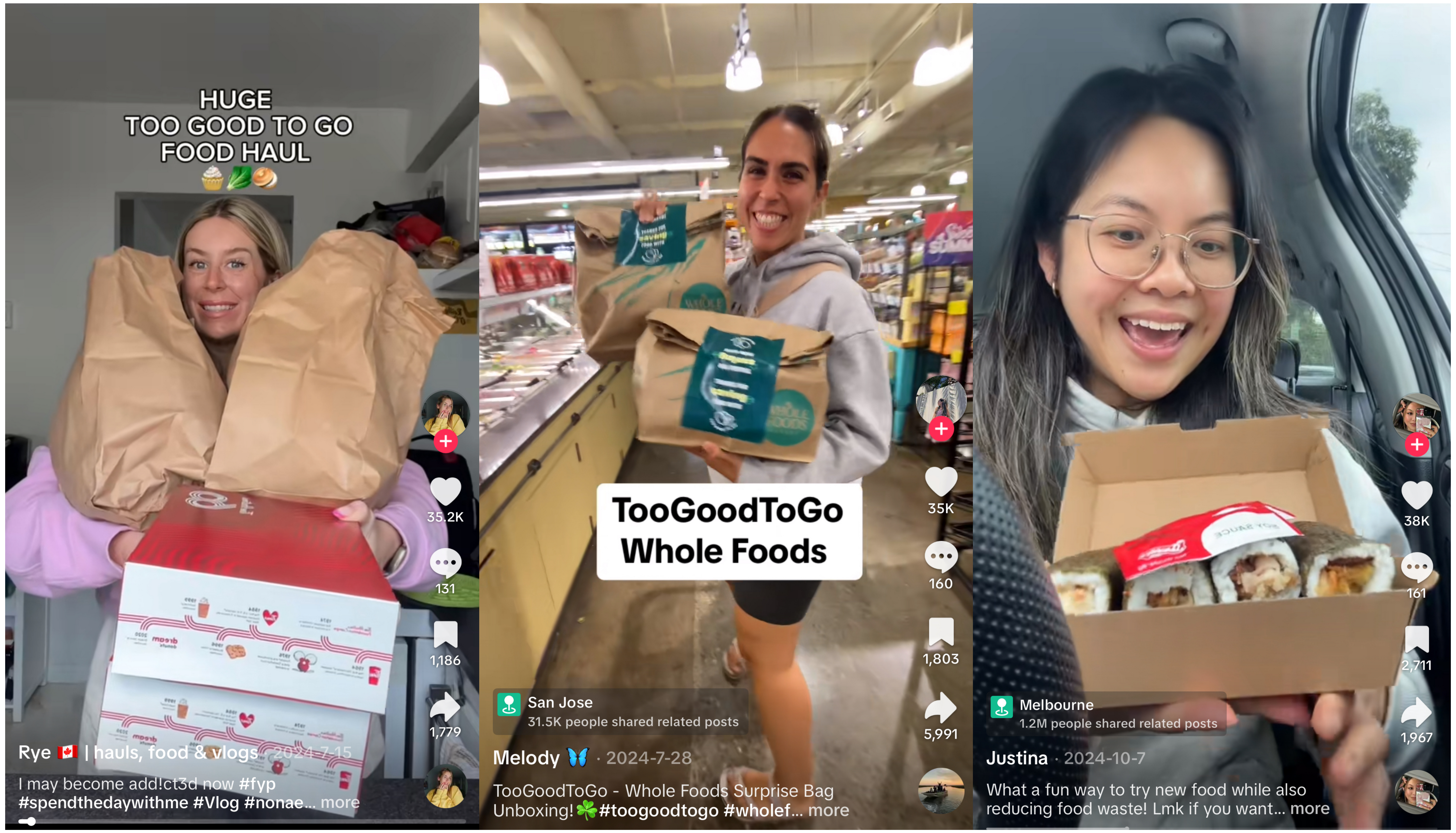

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Too Good To Go

“ Users purchase Surprise Bags filled with a mix of surplus food items.

We know that food waste varies on a day-to-day basis, so this is our way of making sure retailers have the flexibility to sell genuine surplus - whatever that ends up being.

Surprise Bags are sold at a reduced price of the contents’ original retail value, typically priced at approximately 25 to 50% of the original retail value. ”

Workflow

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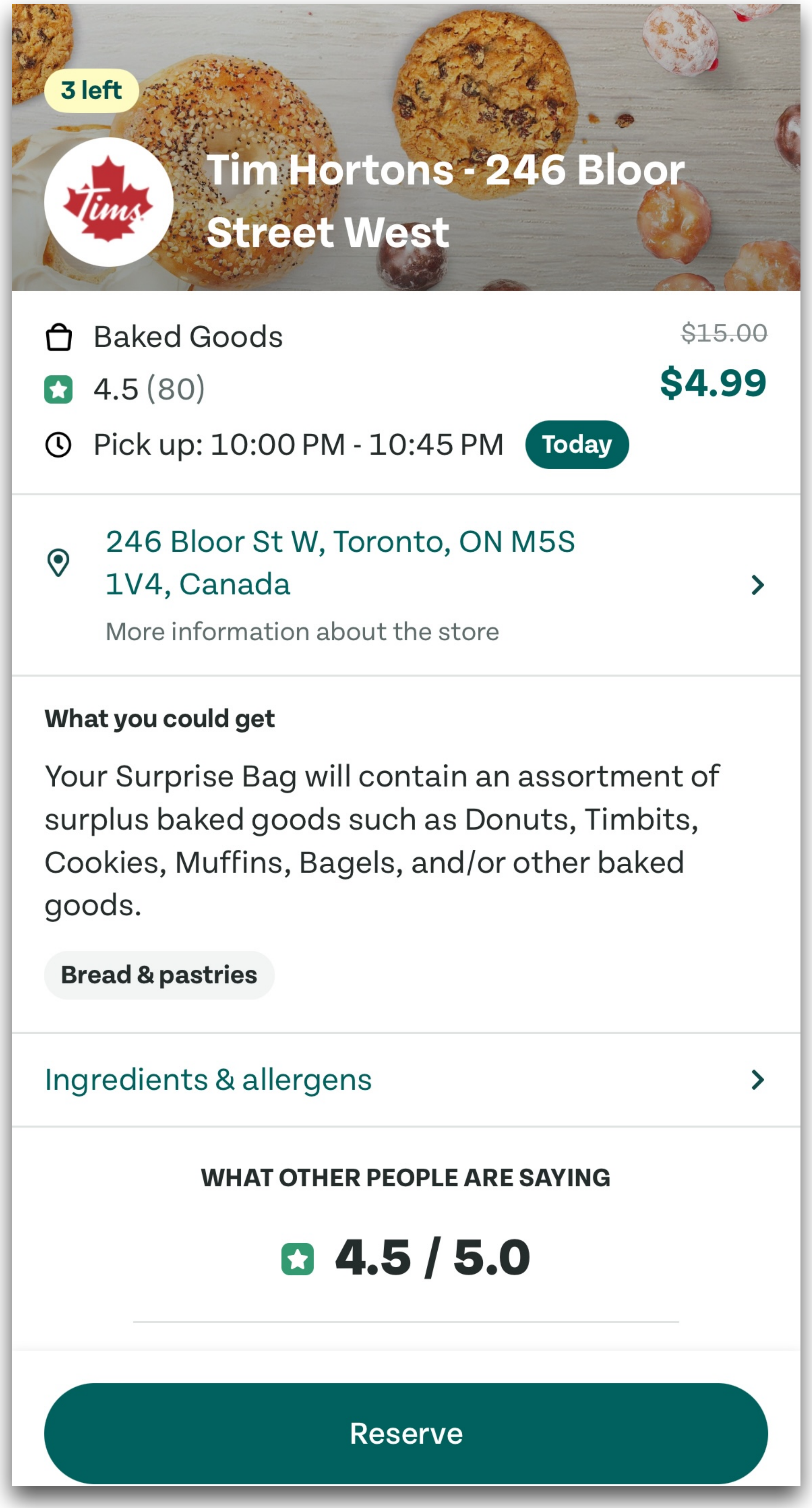
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Tim Hortons - 246 Bloor Street West
Baked Goods
$15.00
$4.99
4.5 (80)
Pick up: 10:00 PM - 10:45 PM
Today
246 Bloor St W, Toronto, ON M5S 1V4, Canada
More information about the store
What you could get
Your Surprise Bag will contain an assortment of surplus baked goods such as Donuts, Timbits, Cookies, Muffins, Bagels, and/or other baked goods.
Bread & pastries
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WHAT OTHER PEOPLE ARE SAYING
4.5 / 5.0
Reserve

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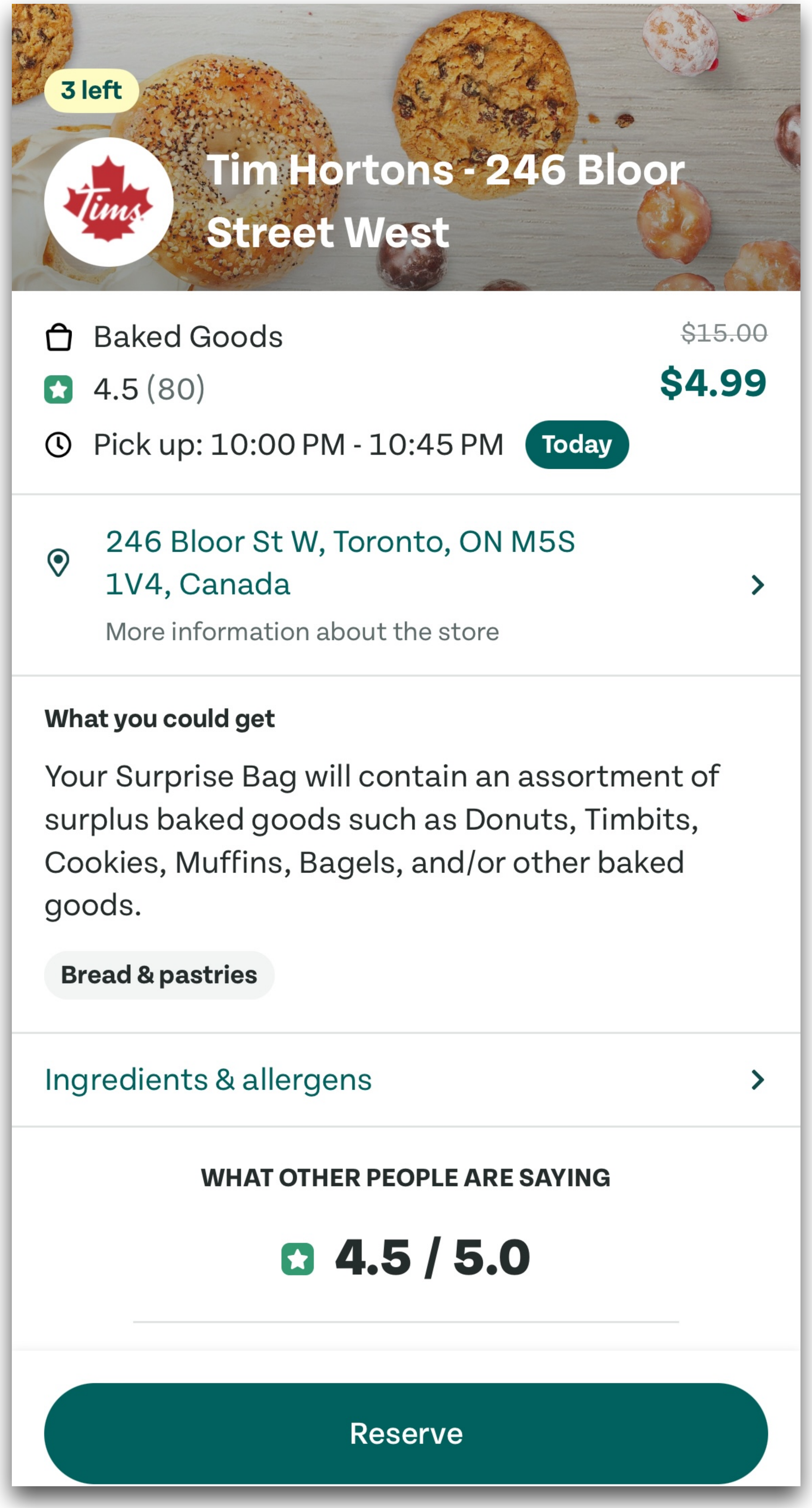
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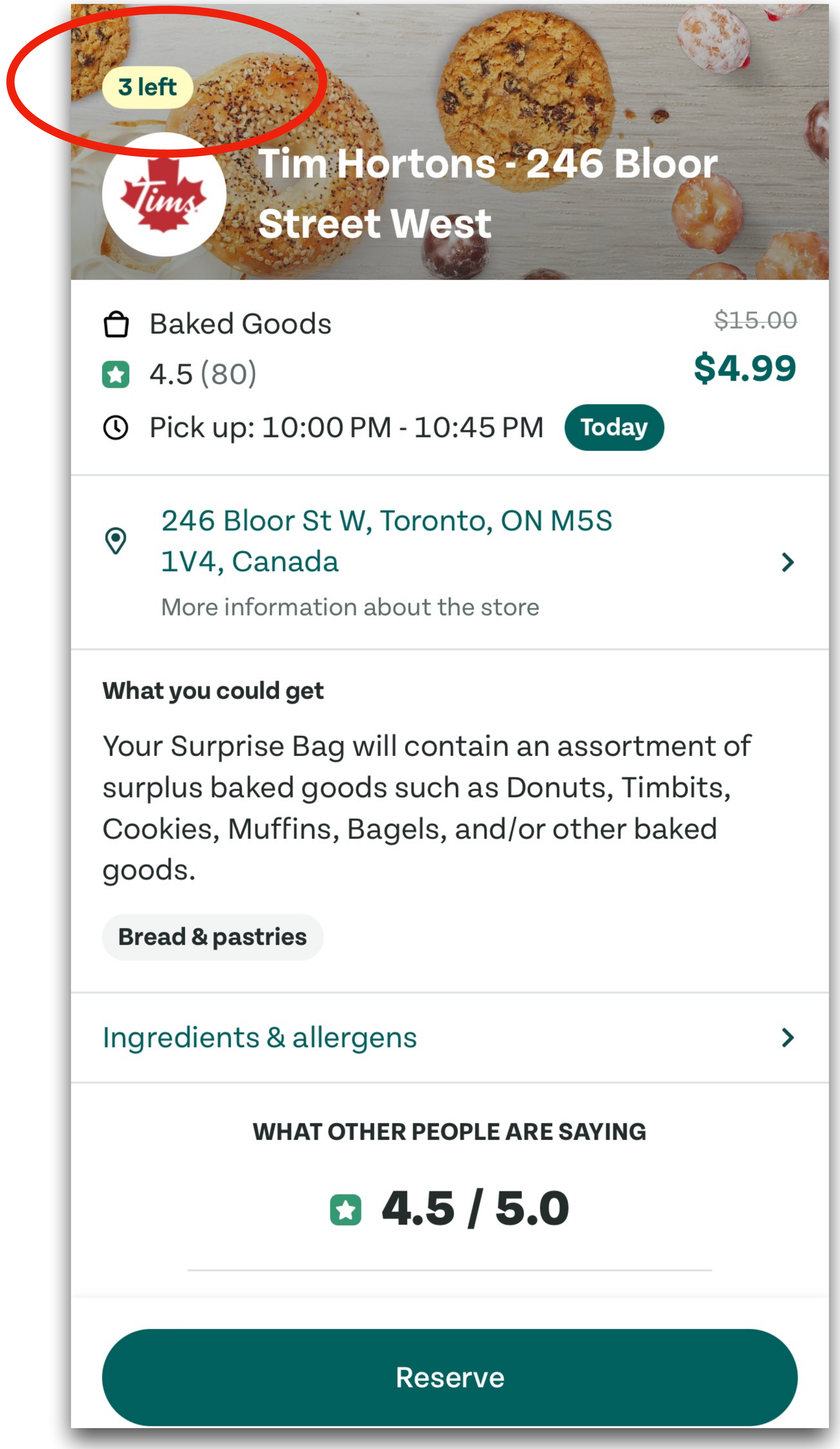

Participating businesses list surprise bags of surplus foods

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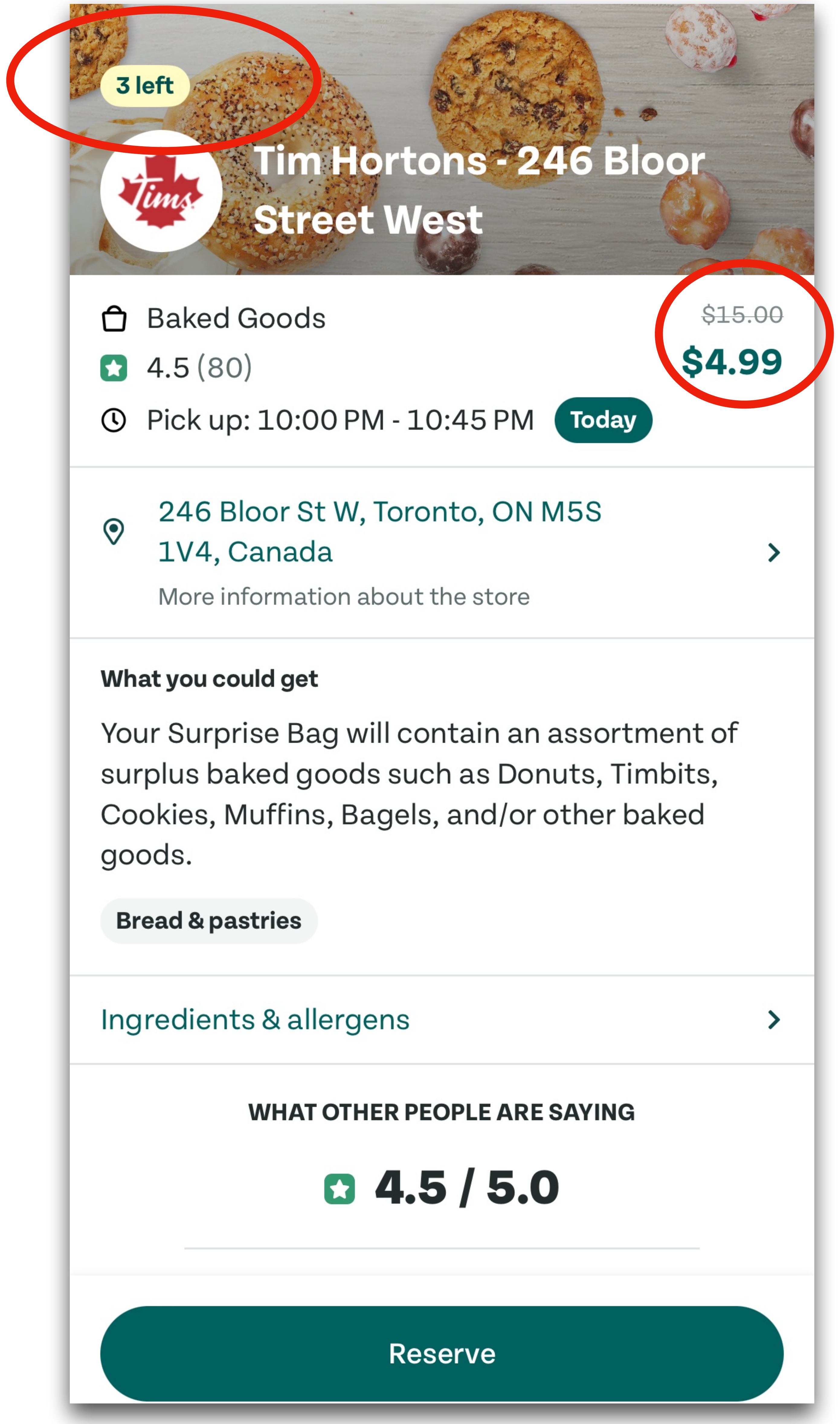

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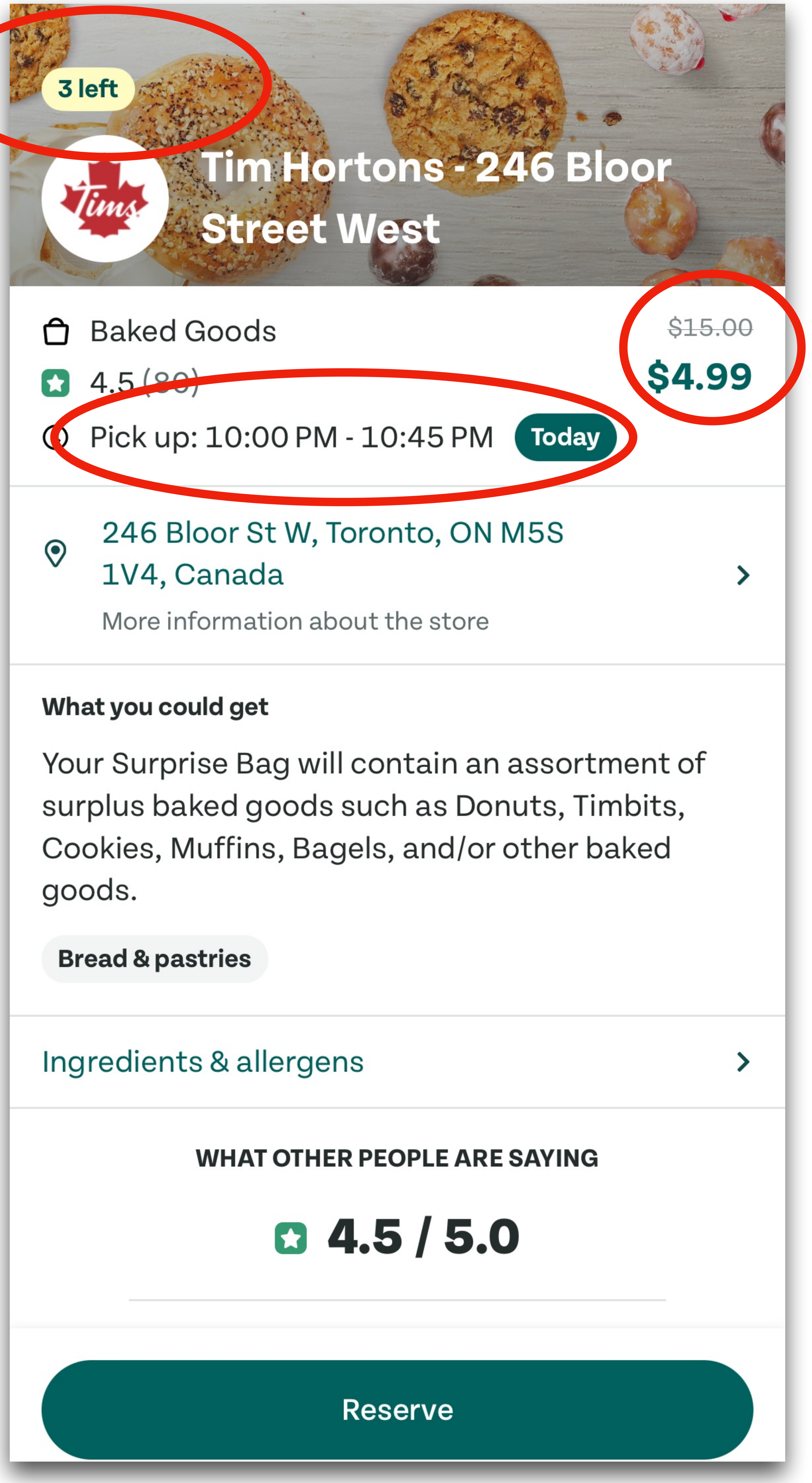

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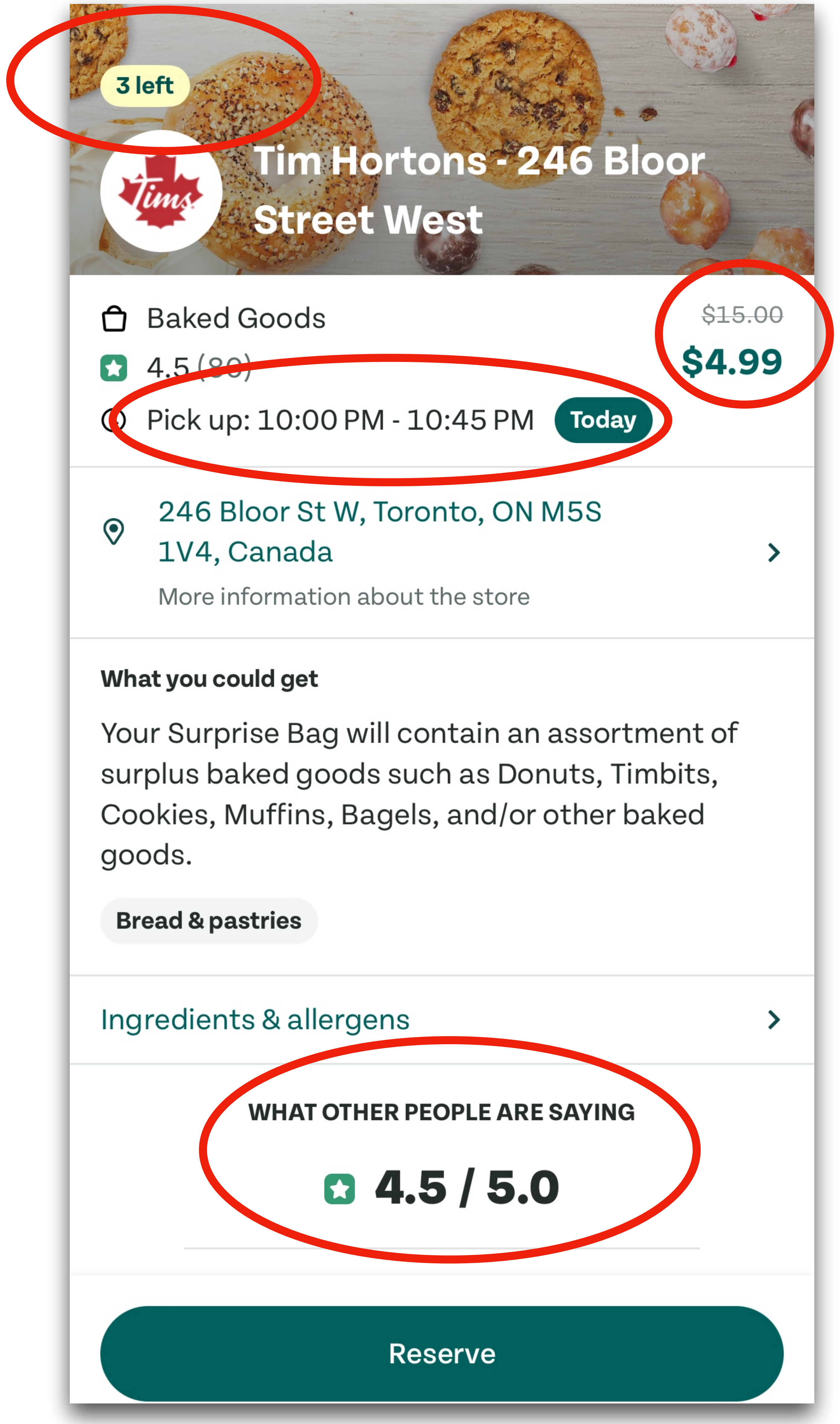

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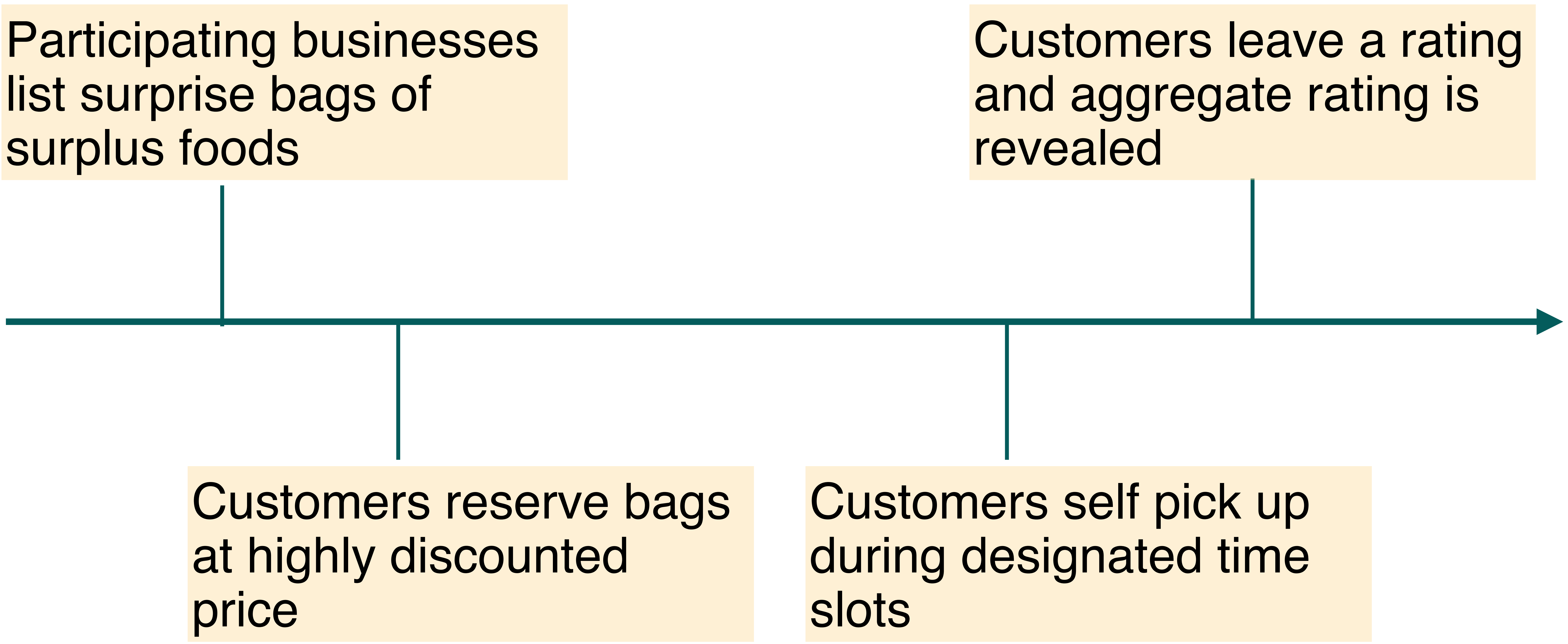

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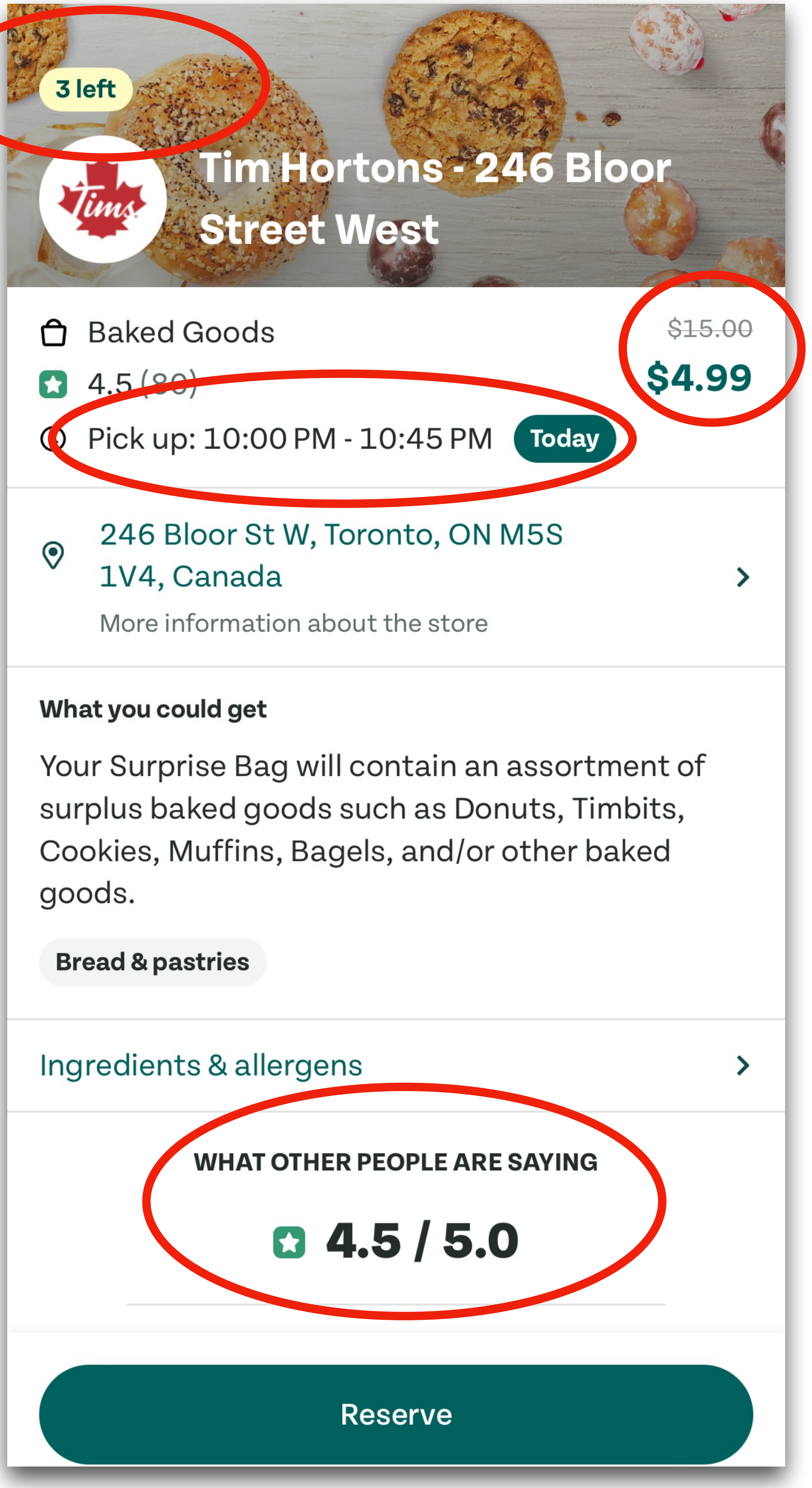

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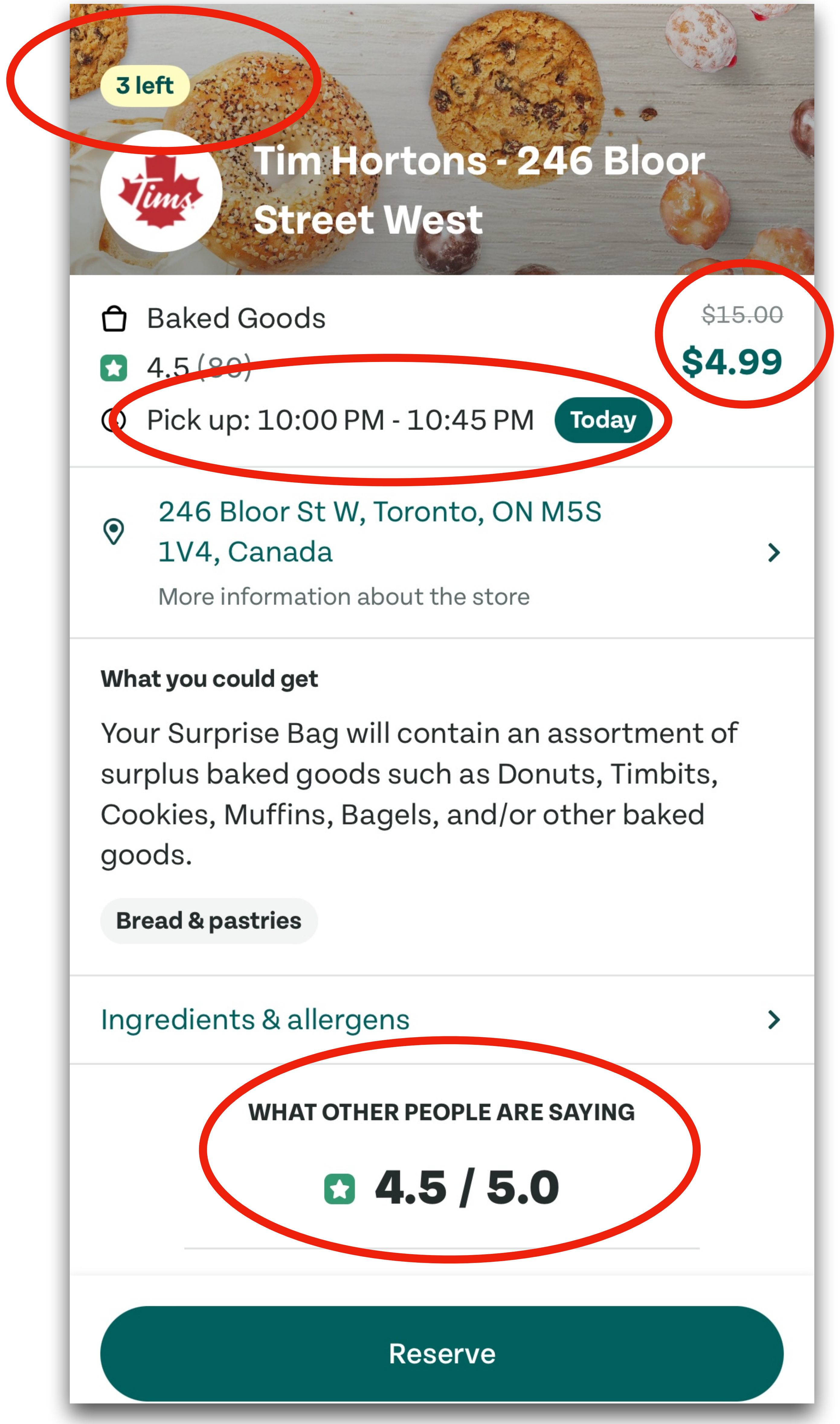

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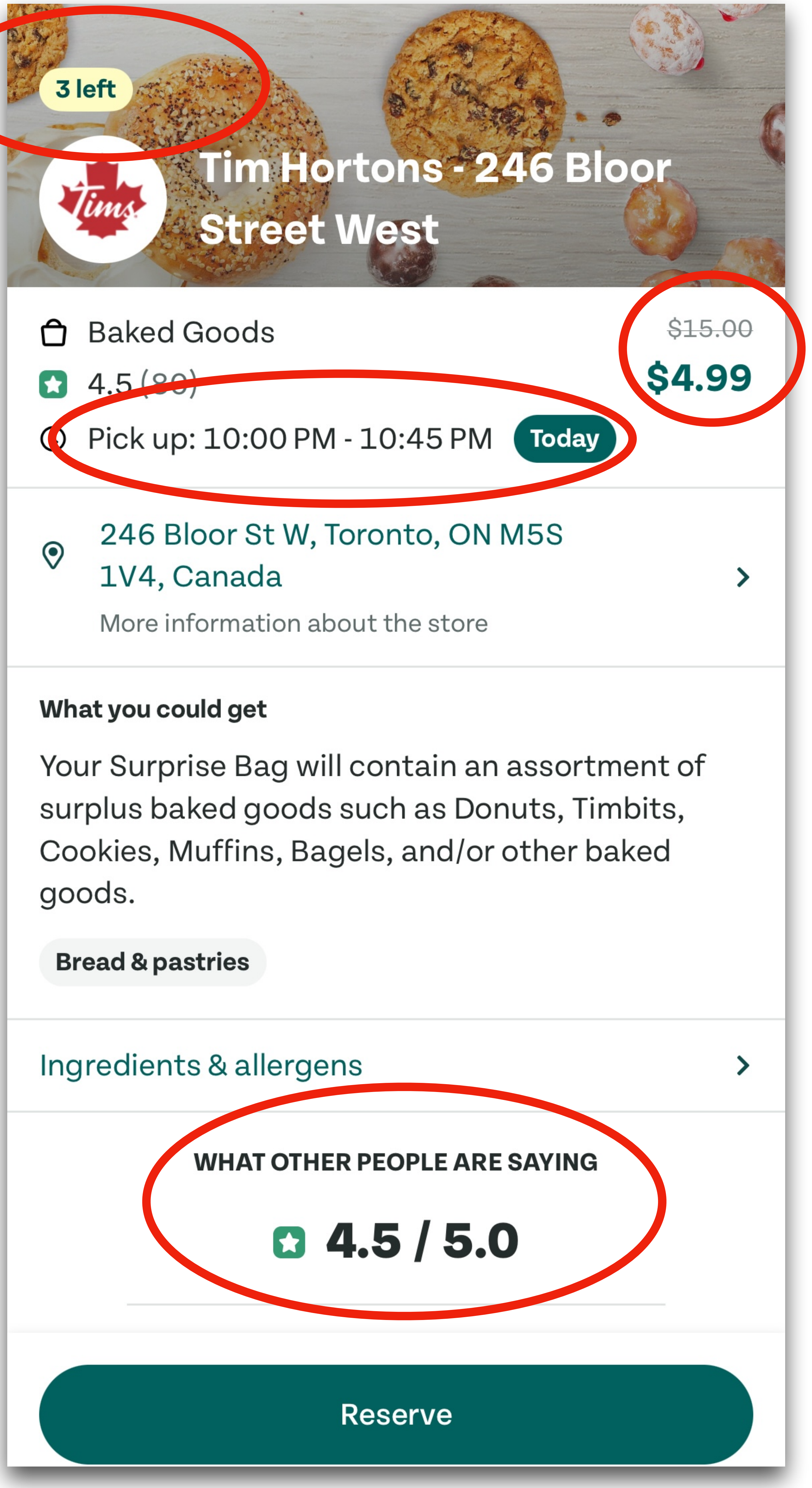

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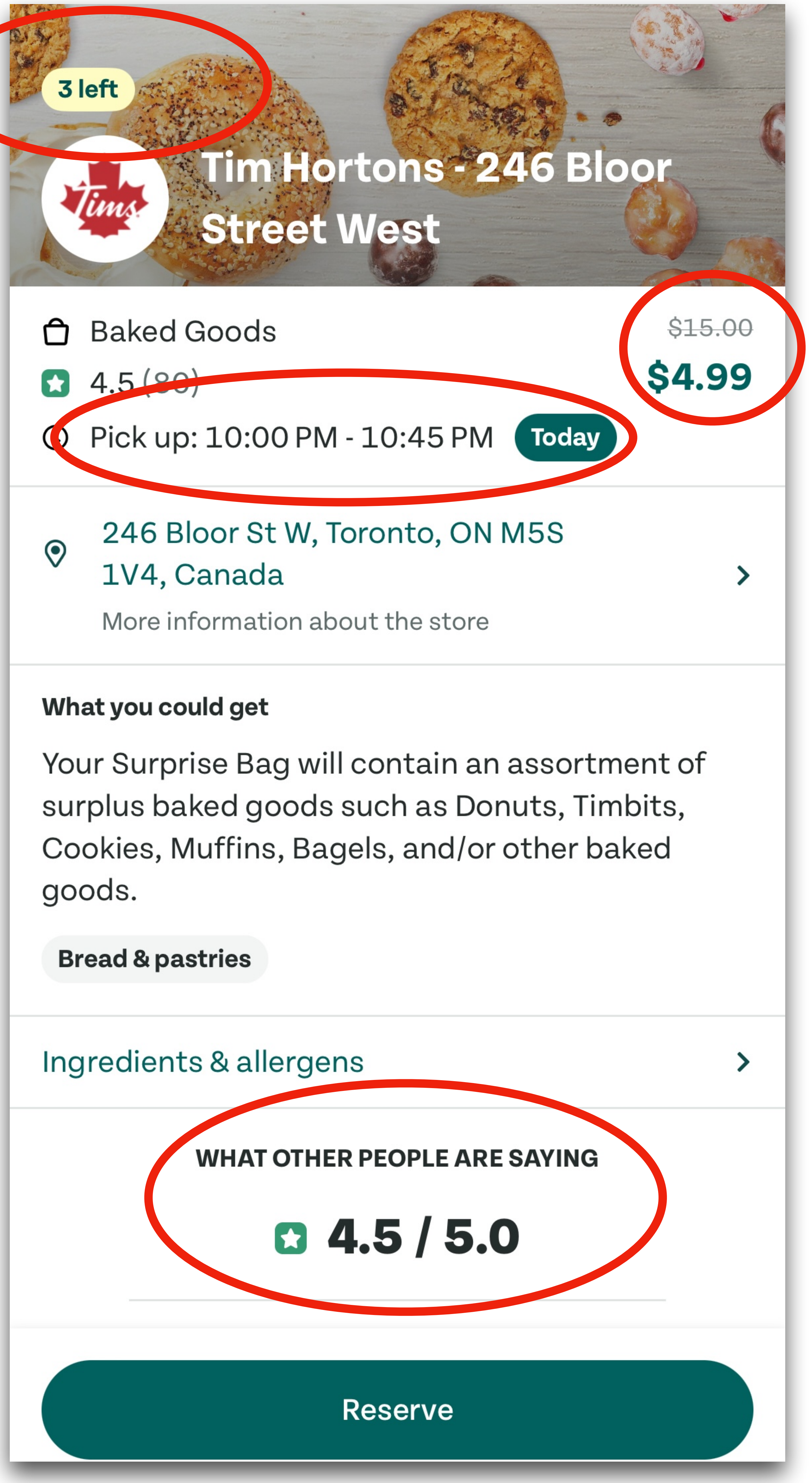

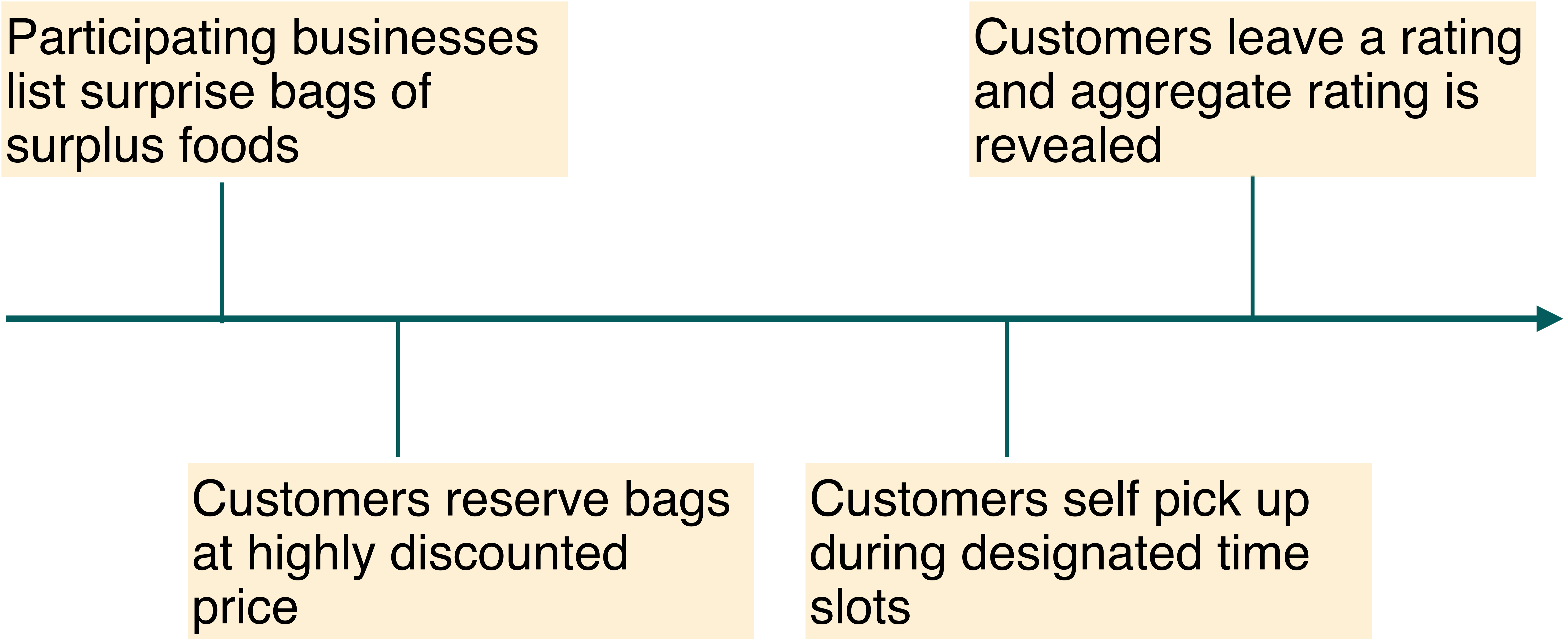

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- **Commission-Based Fee:** The platform charges a commission for each bag sold

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- **Allocate content across surprise bags**
  - The common approach is to evenly distribute surplus items across all bags, ensuring a similar monetary value. However, this may not always maximize consumer satisfaction

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Our Research Contribution

Supports the development of these systems by exploring optimal bag design strategies that ensure long-term profitability and satisfaction

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- **Innovative surplus food management as of TGTG** *(New and quickly growing!)*

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  - Our difference: We explore store reputation and the trade-offs between short-term profits and long-term reputation building in the context of TGTG

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  - Our paper is one of the first to examine opaque selling to reduce food waste, and the first to explore the optimal dynamic design of probabilistic goods in this context.

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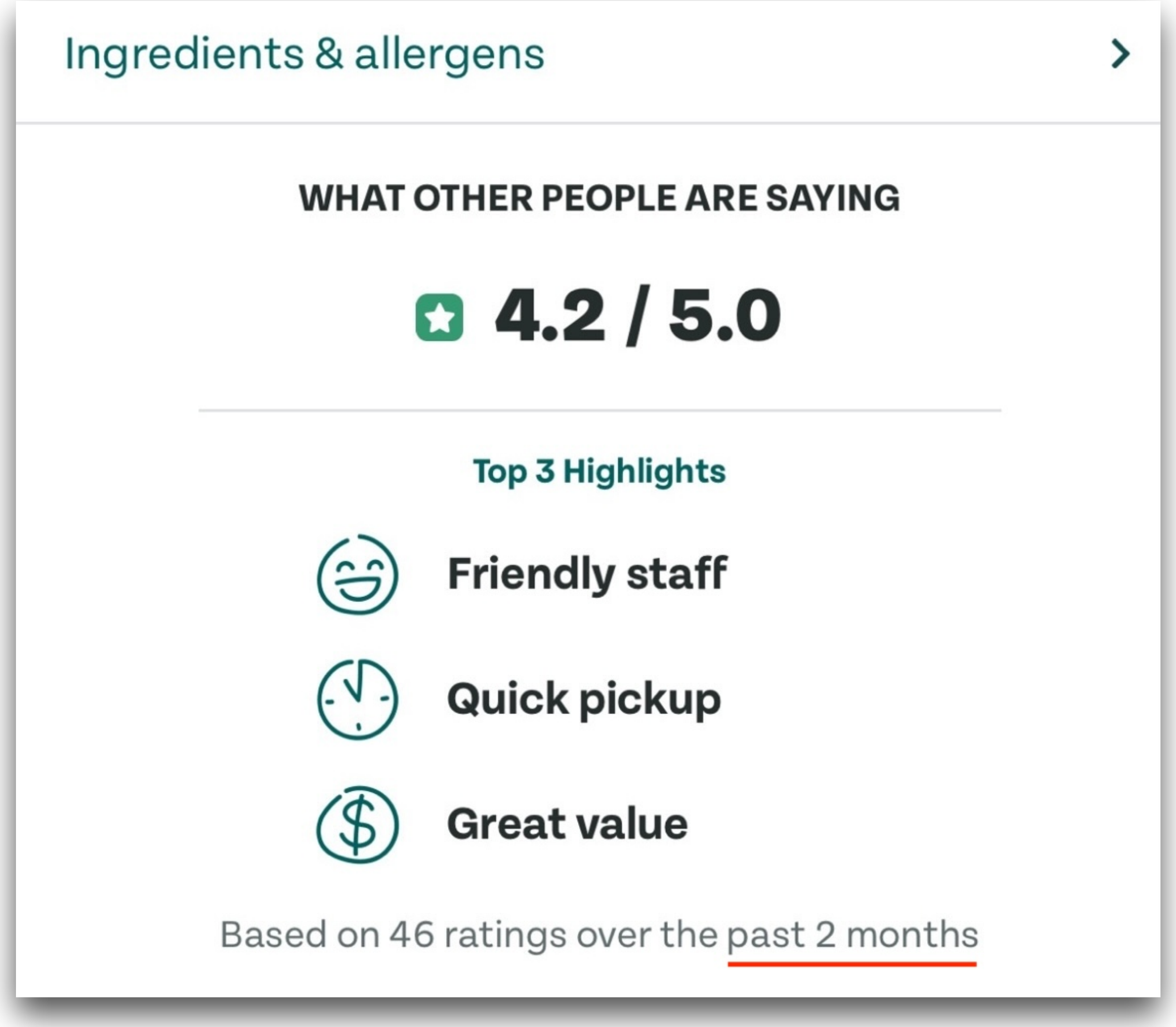

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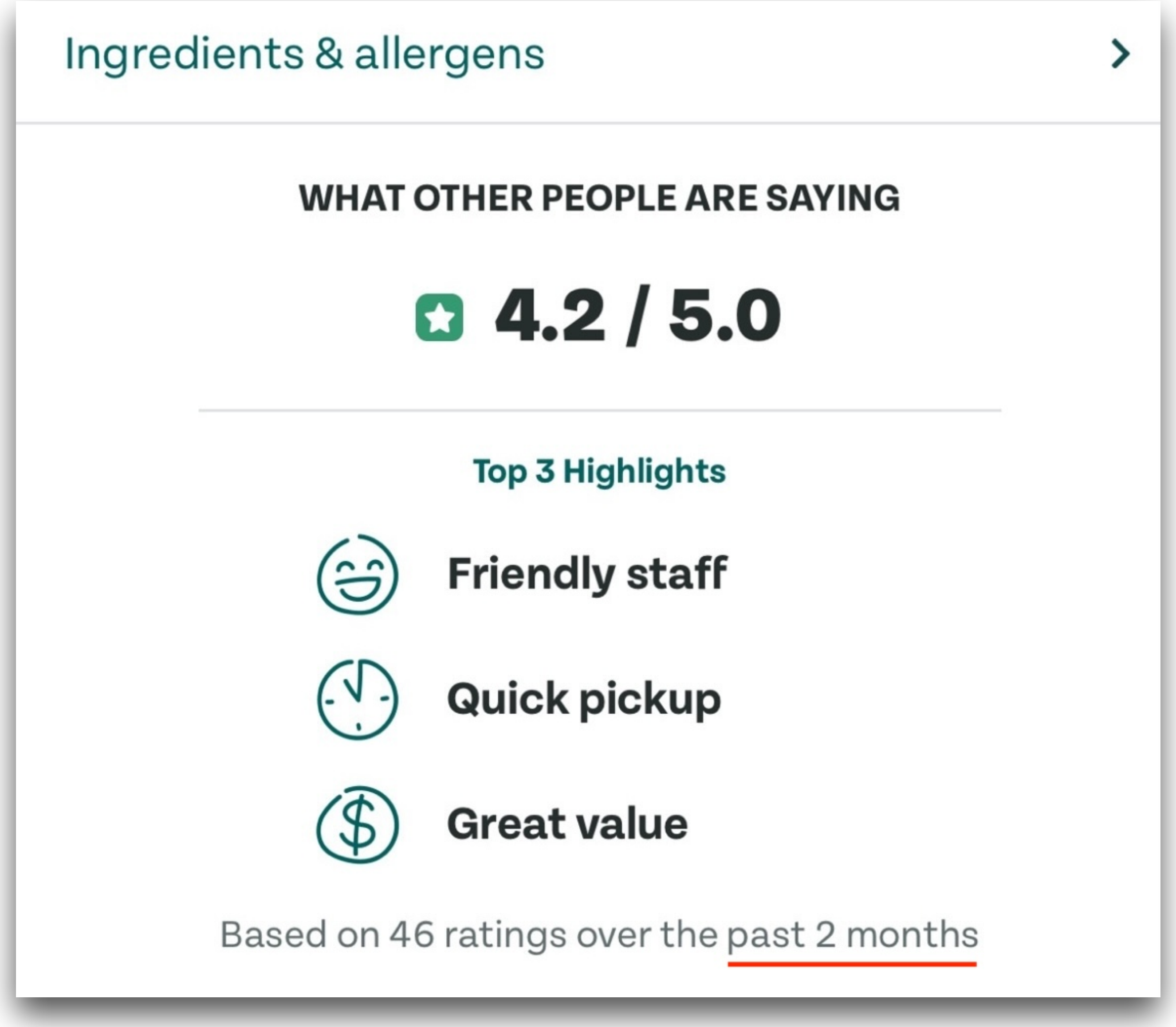

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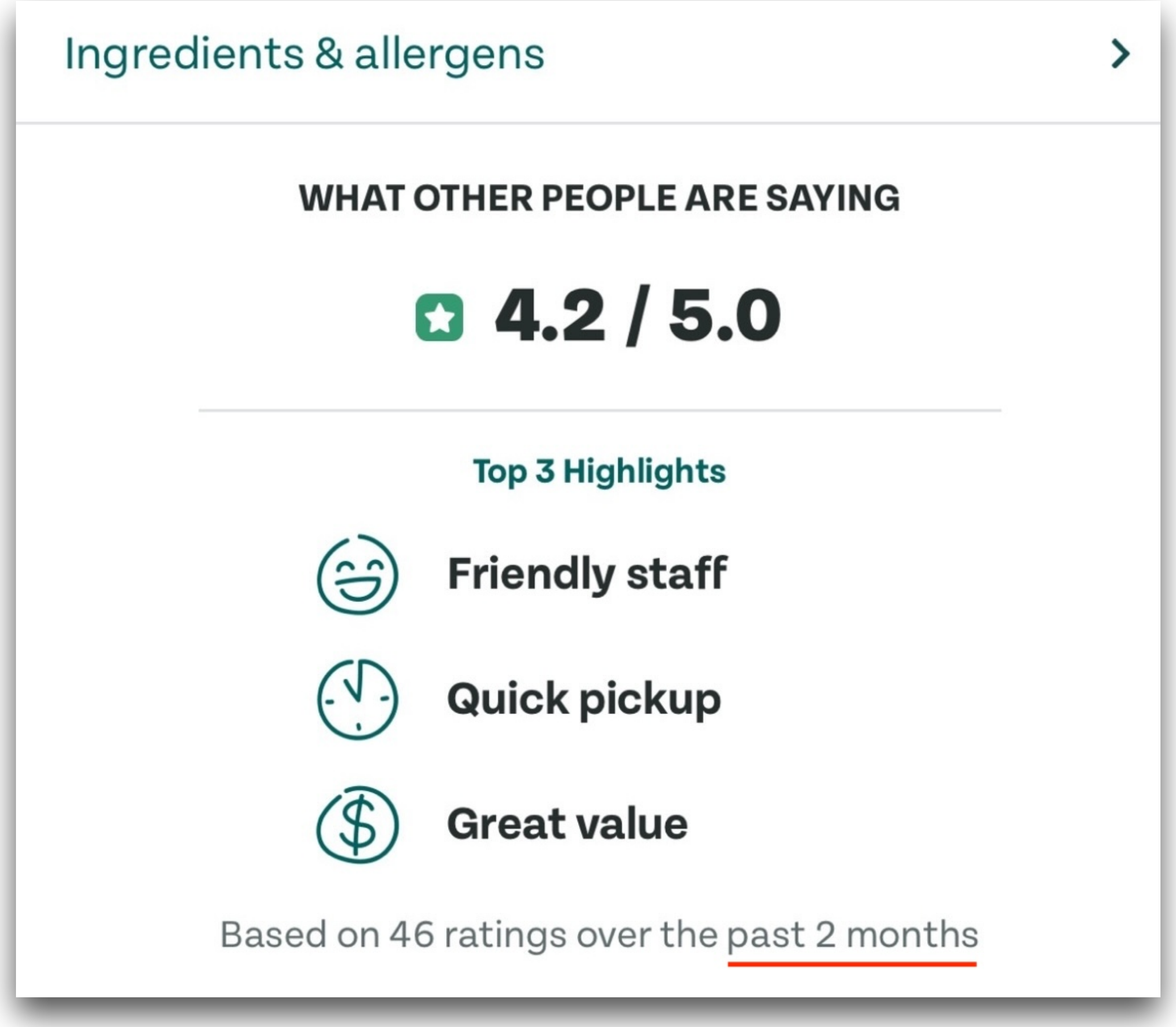

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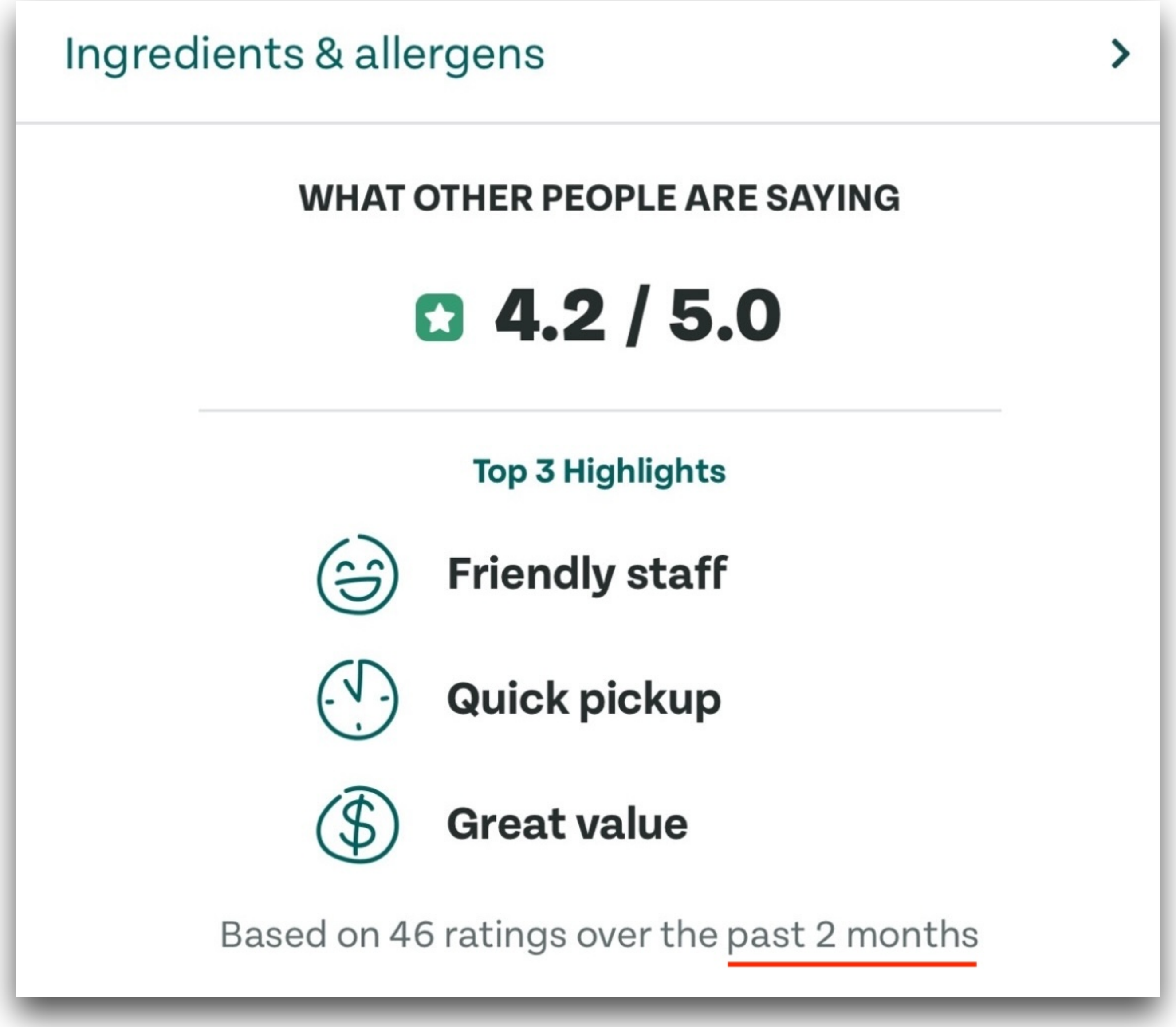

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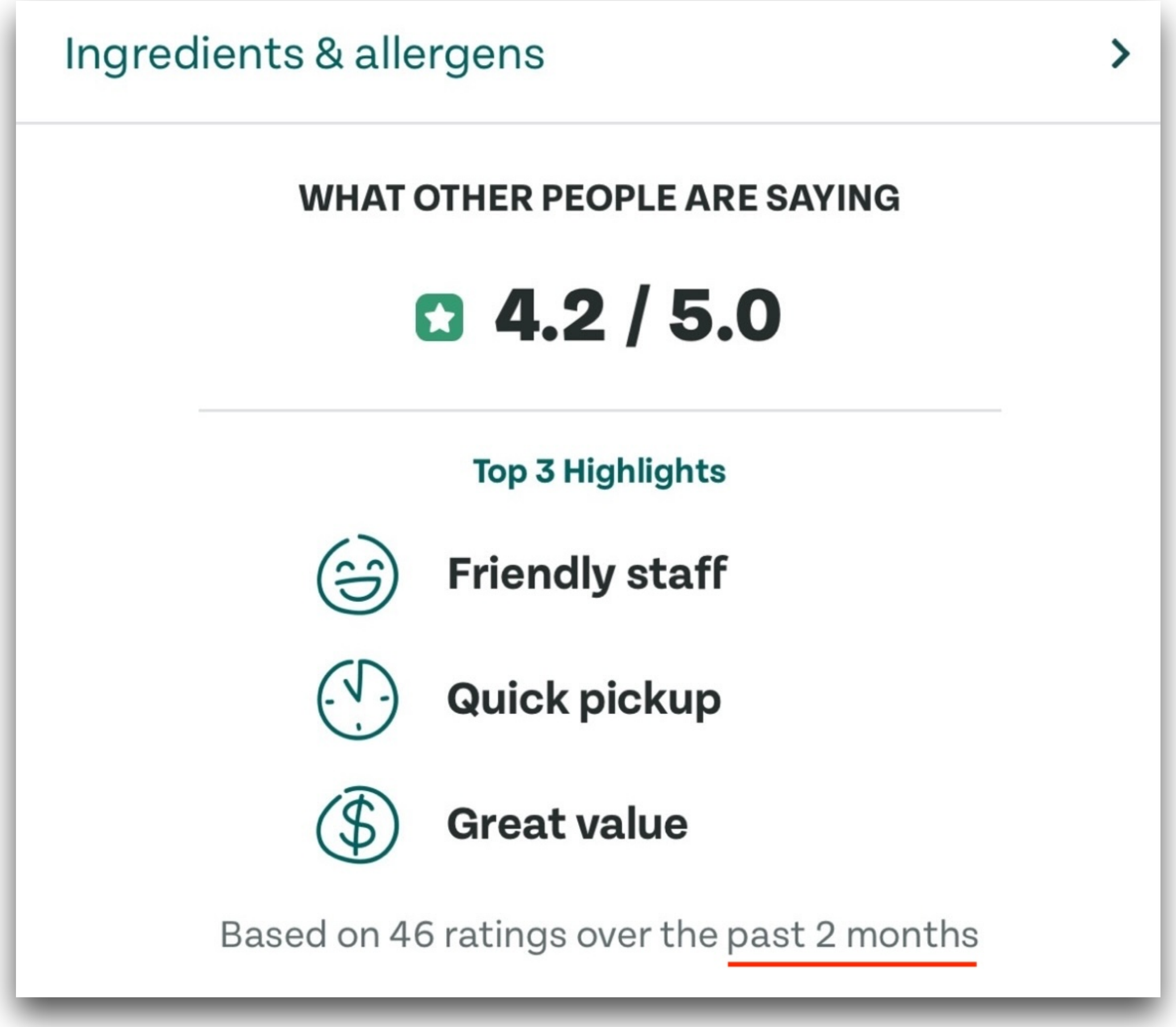

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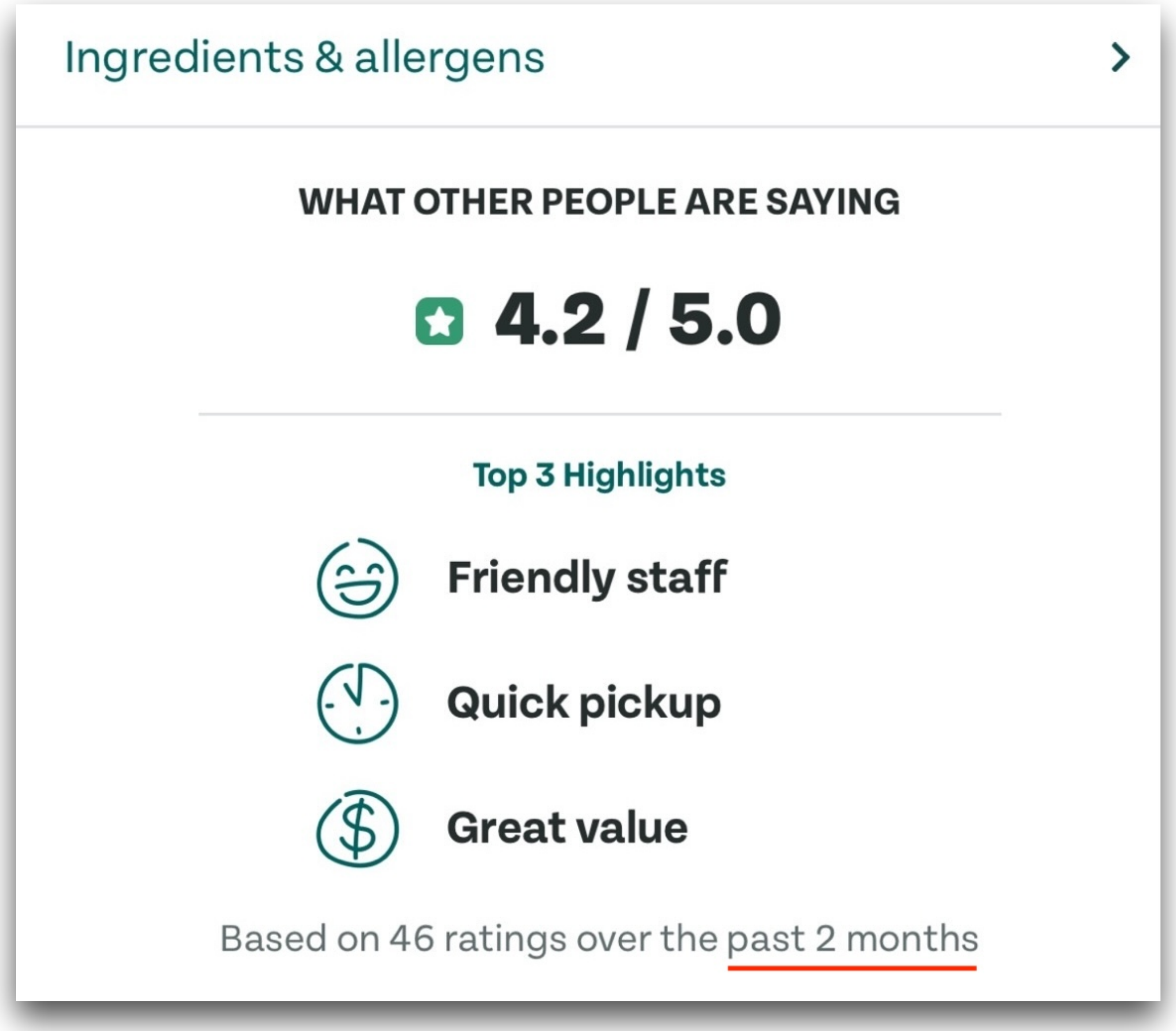

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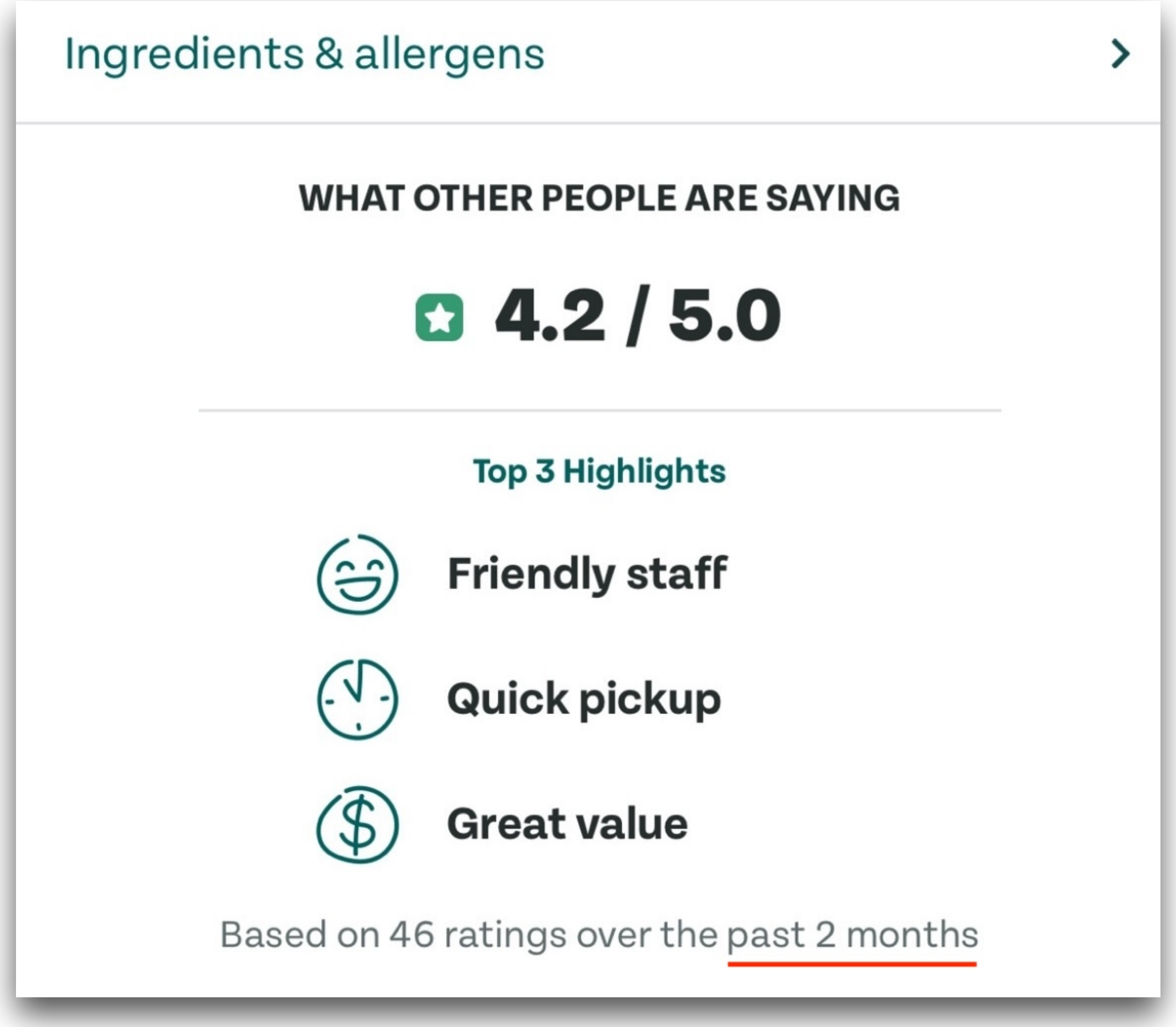

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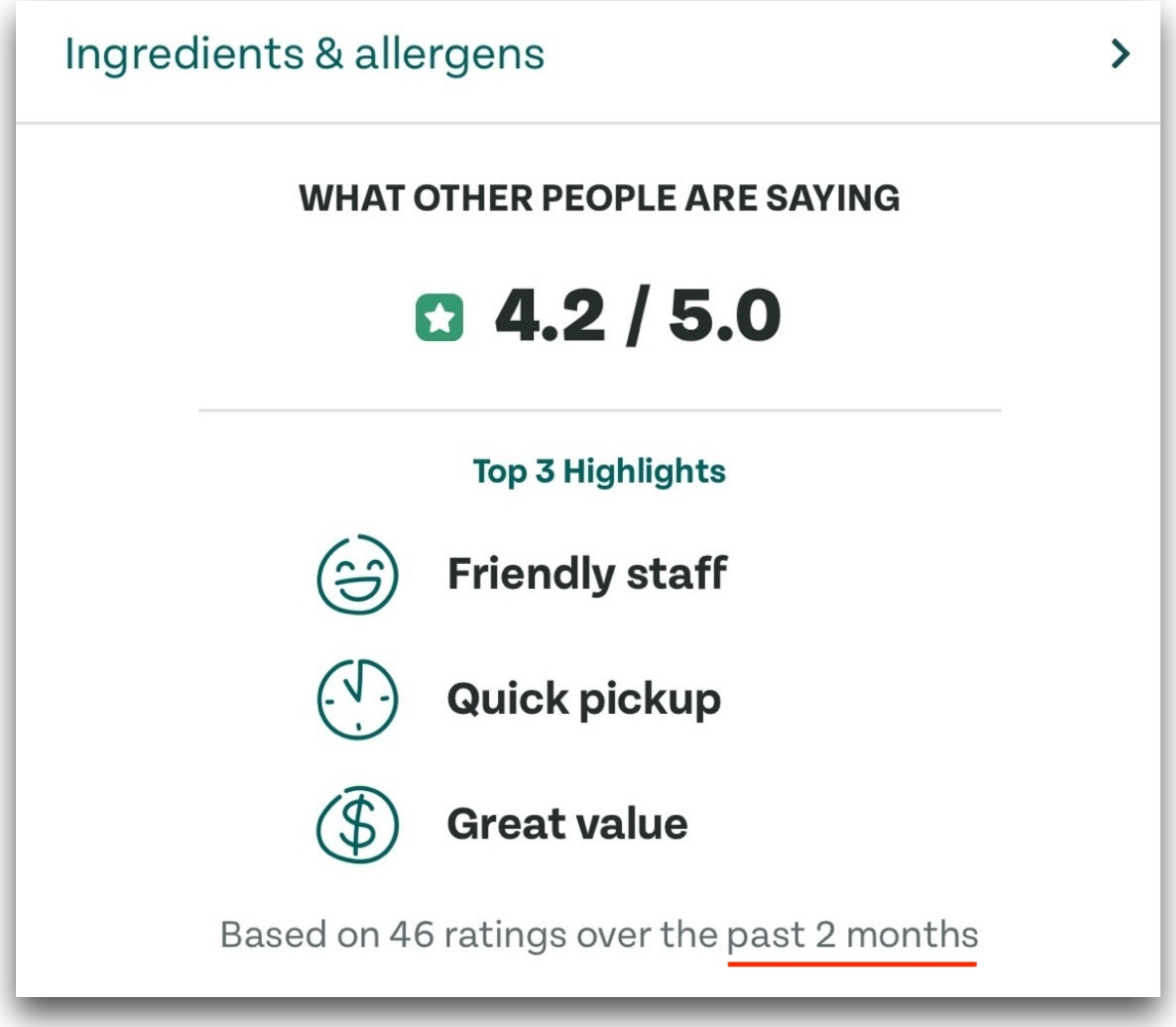

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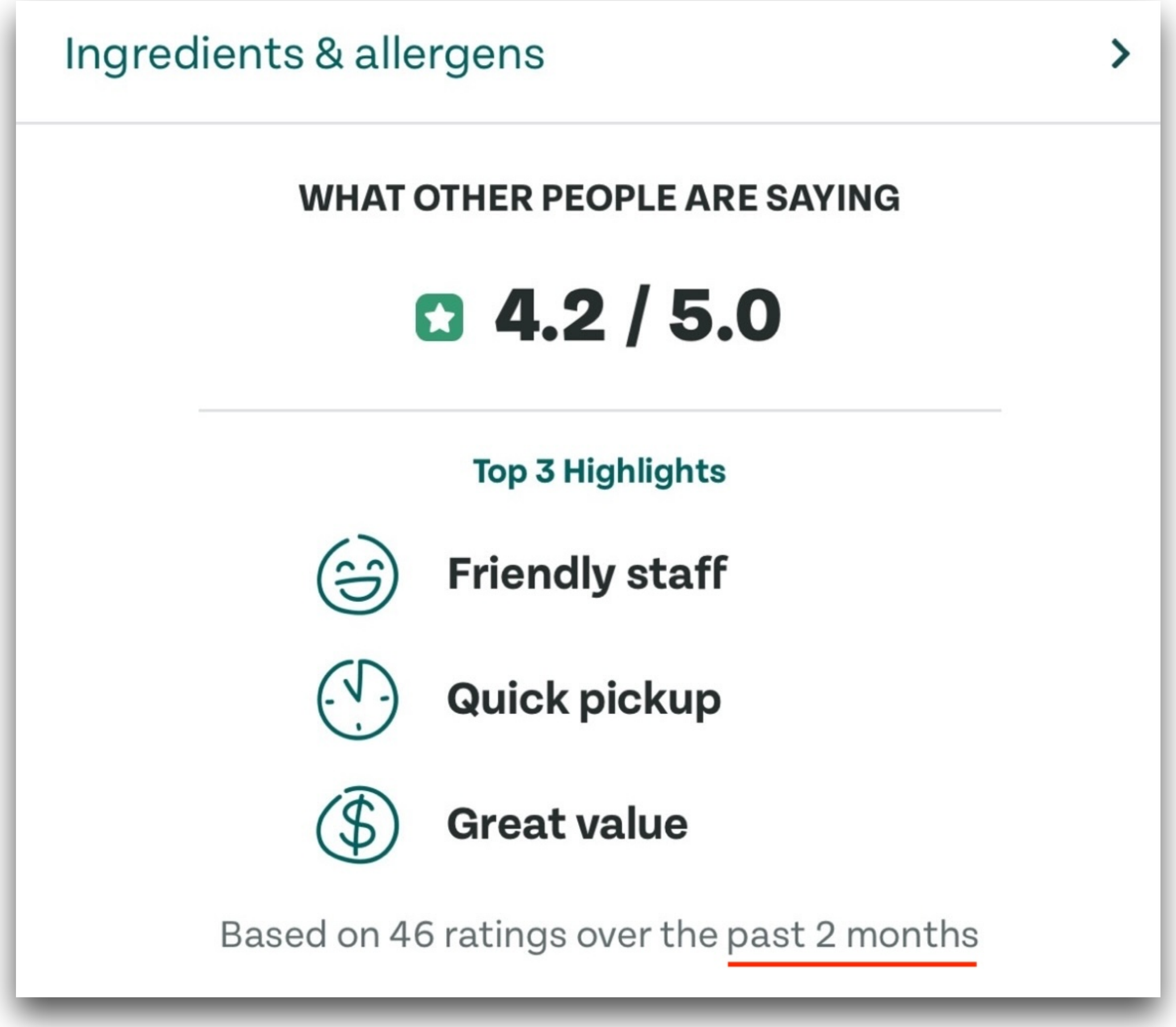

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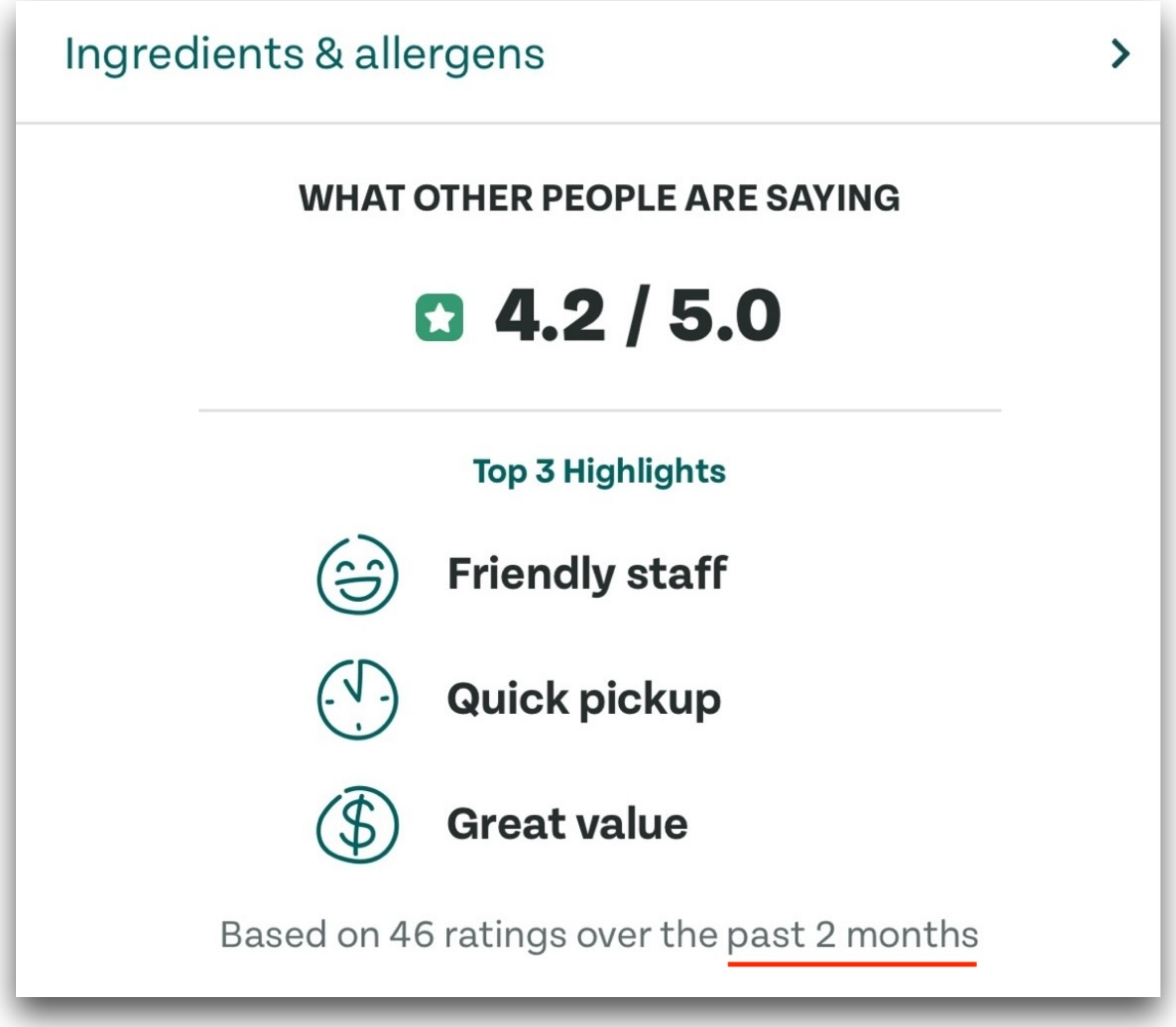

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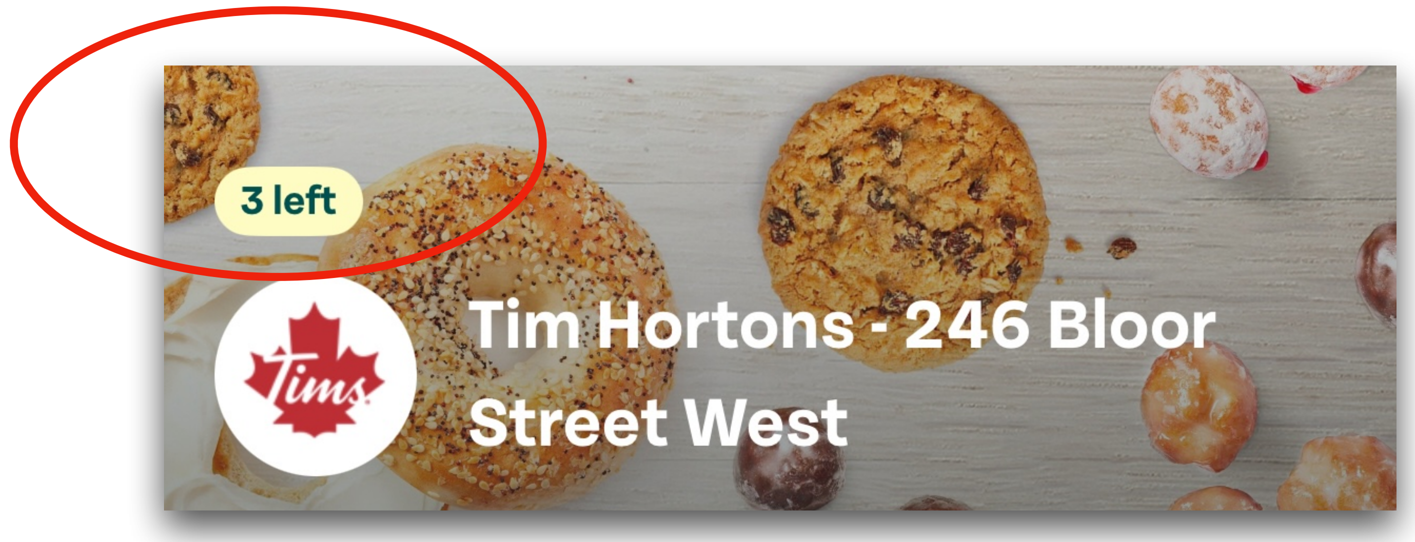

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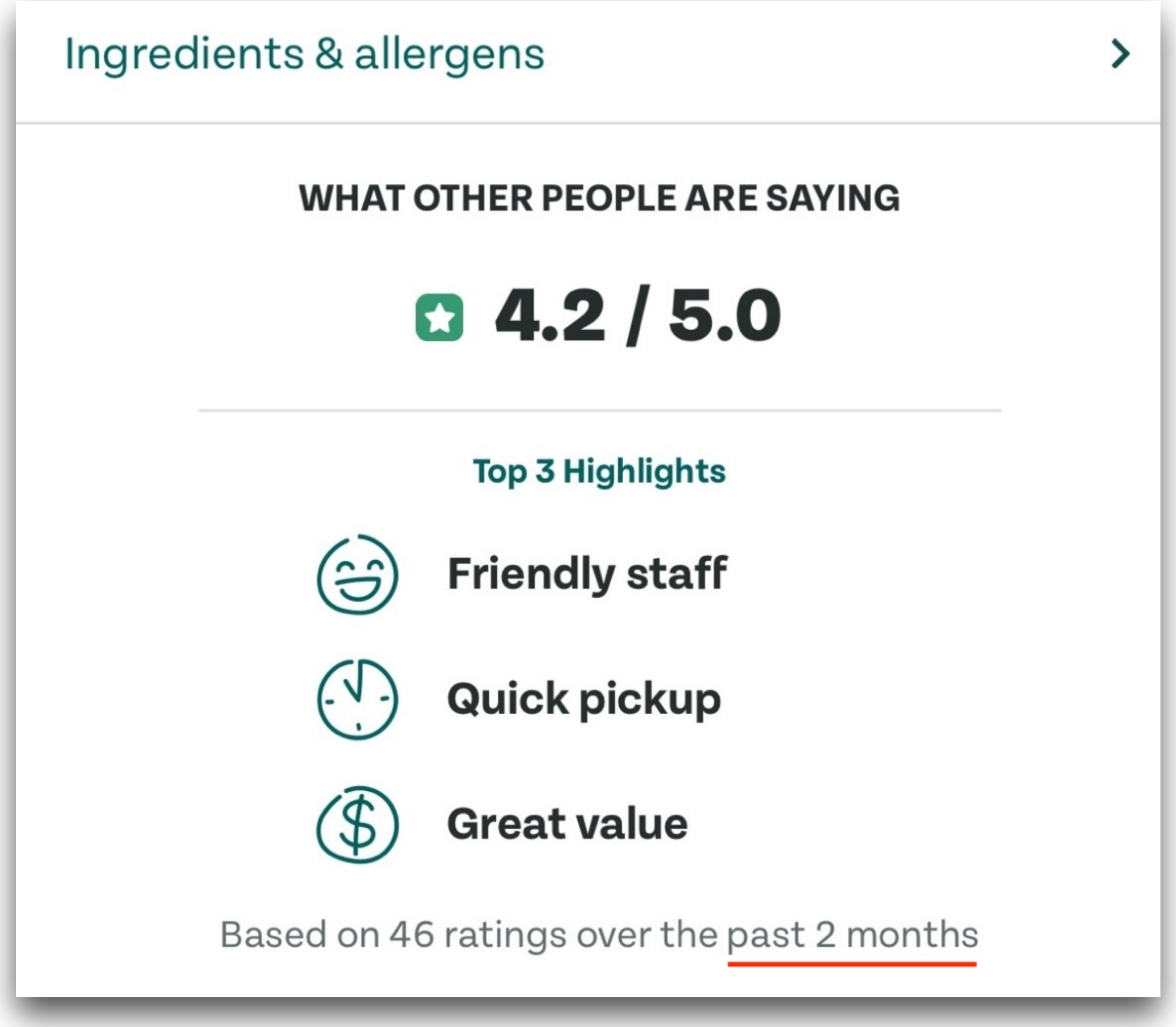

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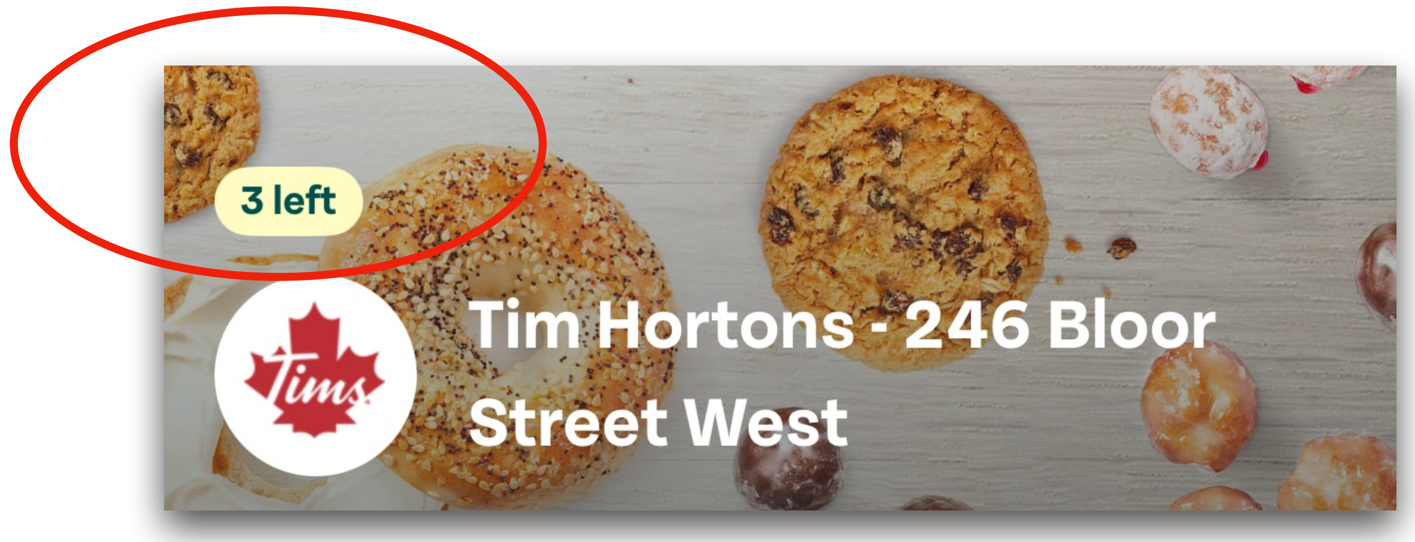

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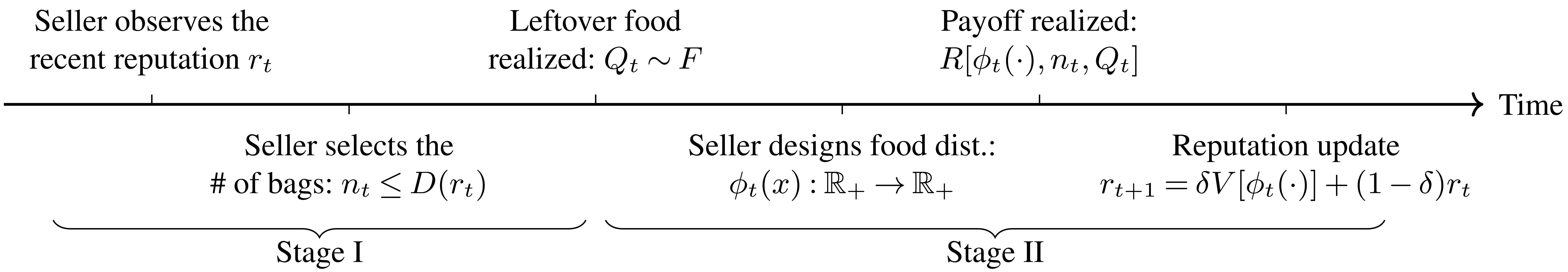

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Leftover food realized: $Q_t \sim F$

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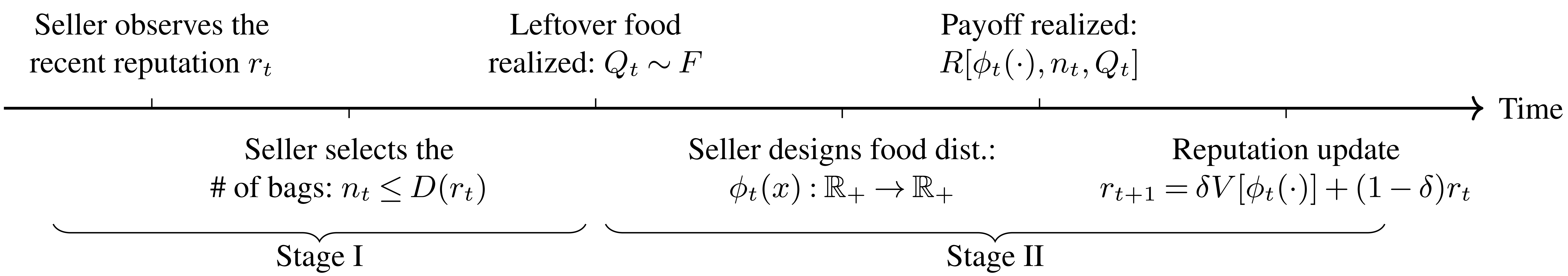

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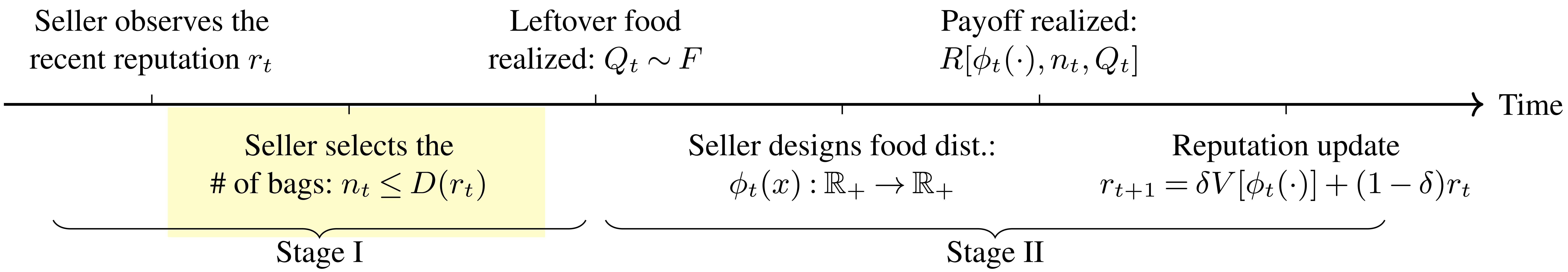

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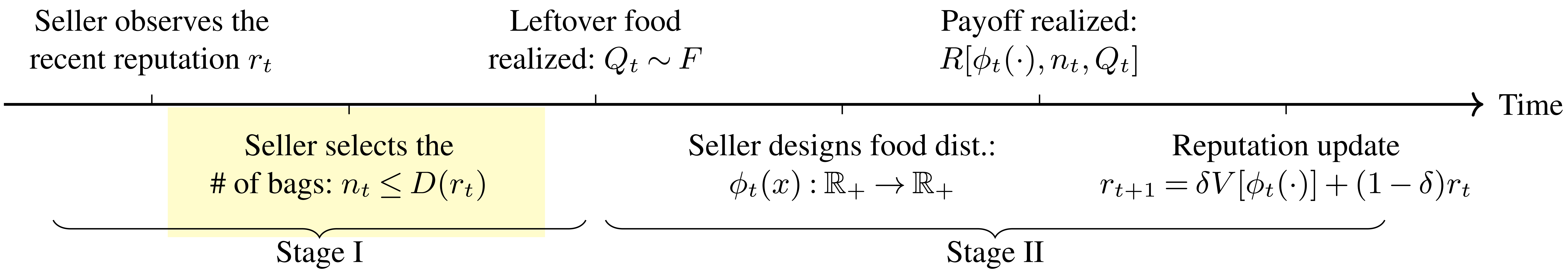

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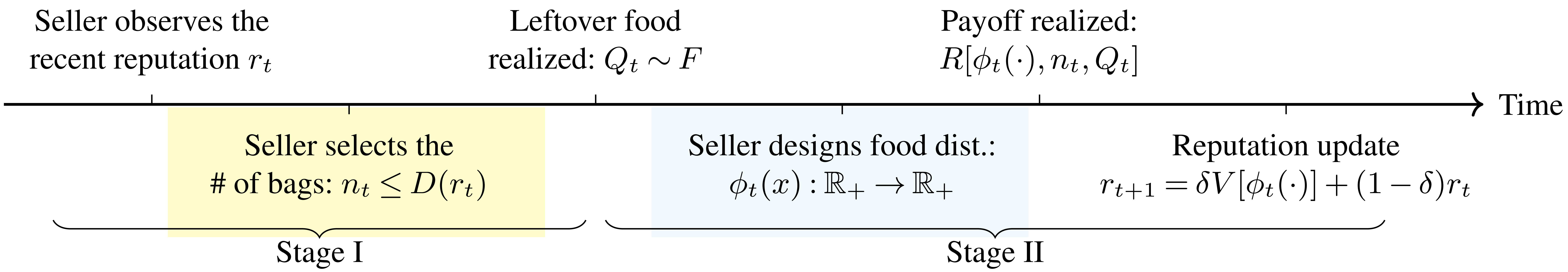

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(Cumulative payoffs)

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subject to $0 \le n_t^\pi \le D(r_t),$ (Upper bound reservation number)

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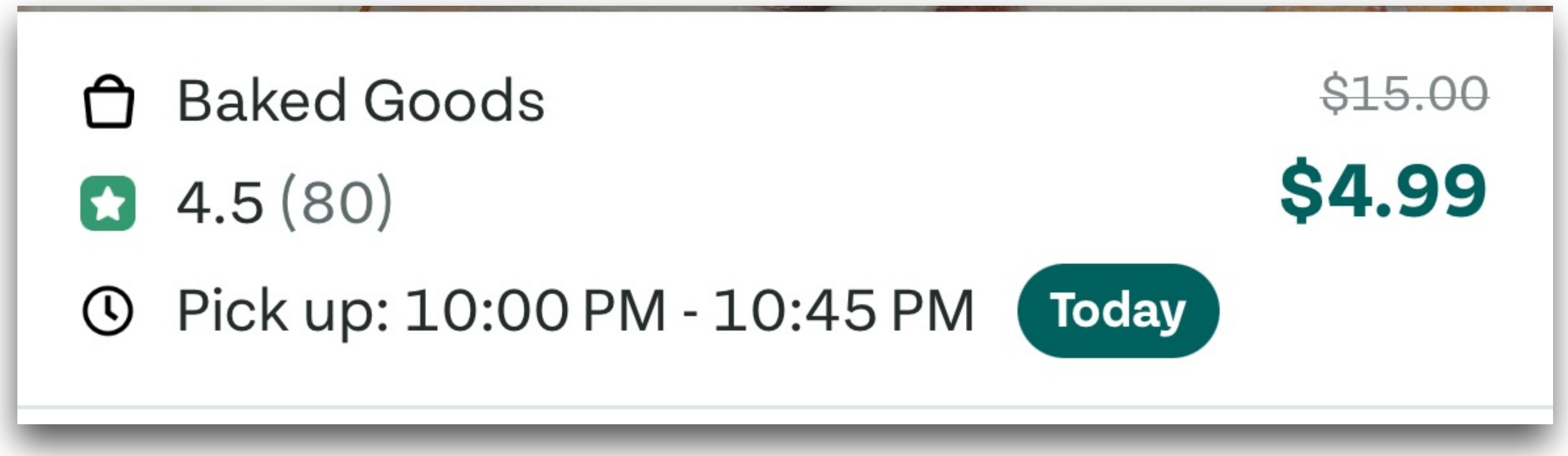

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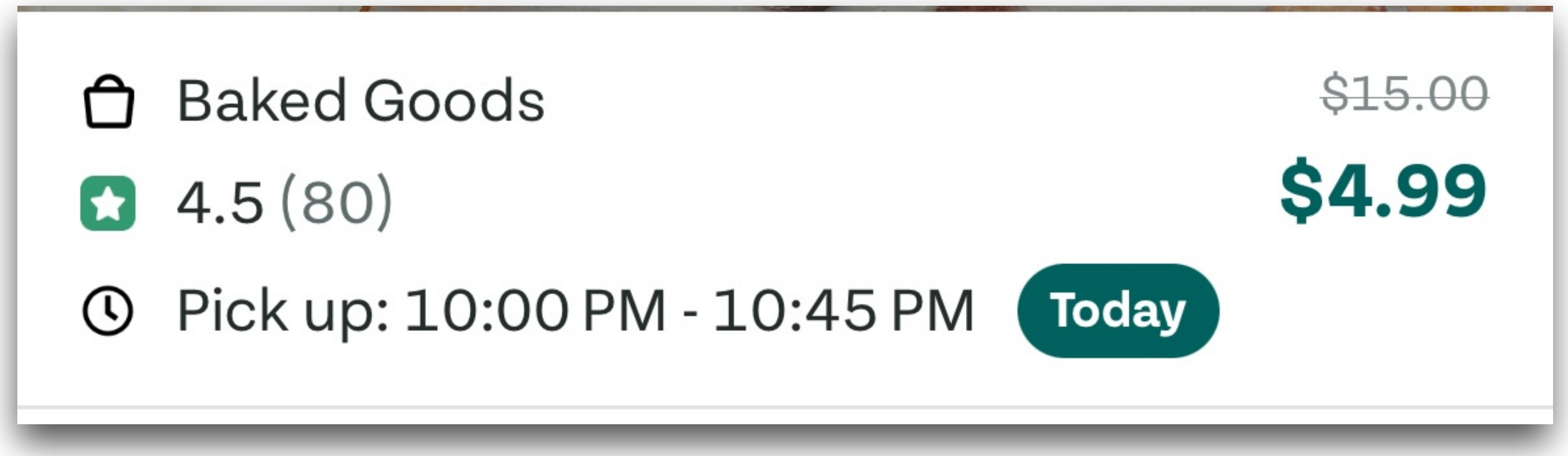

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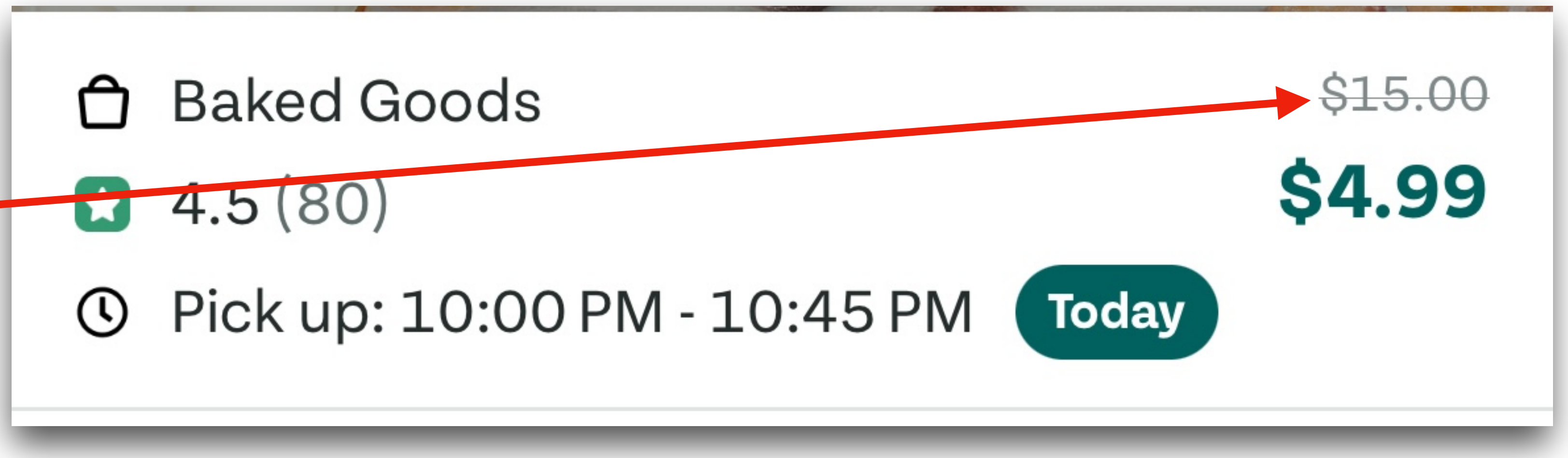

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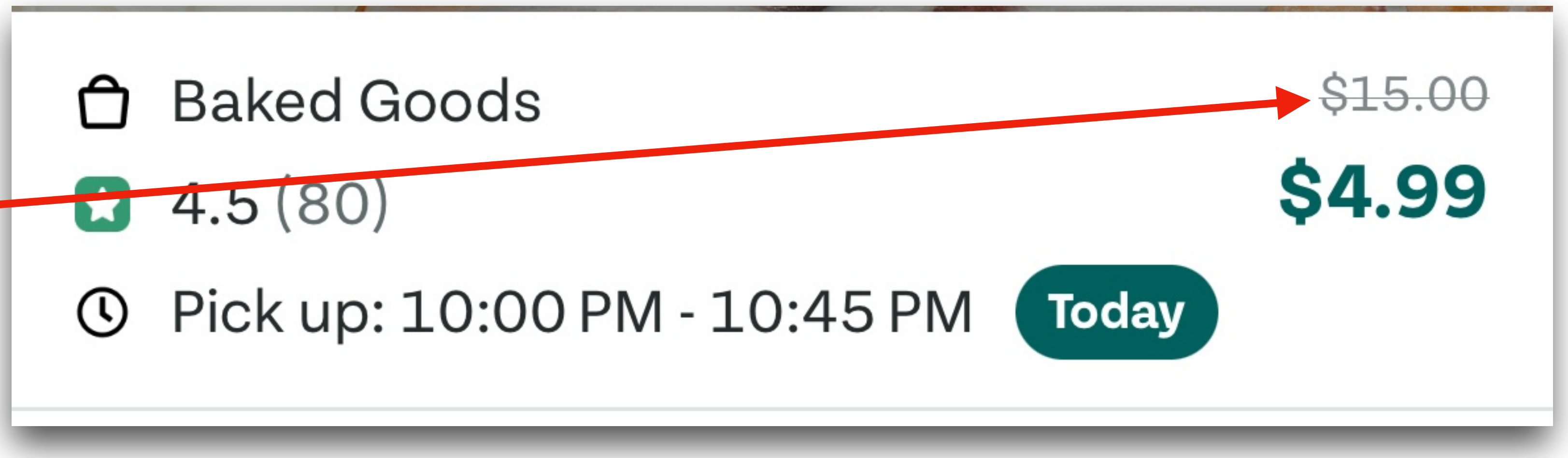

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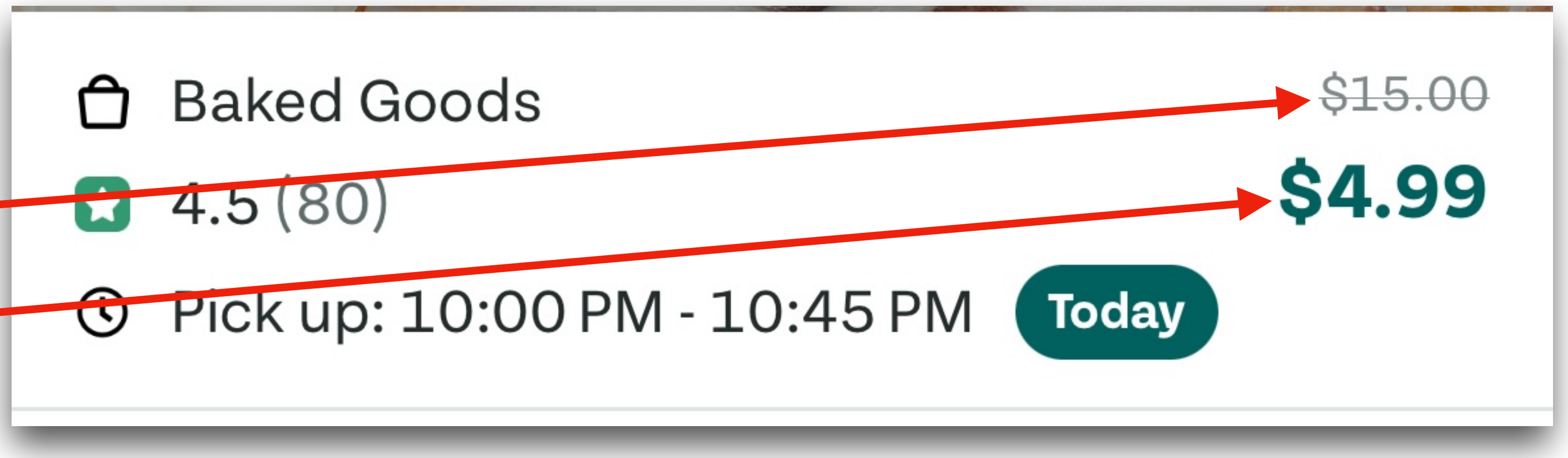

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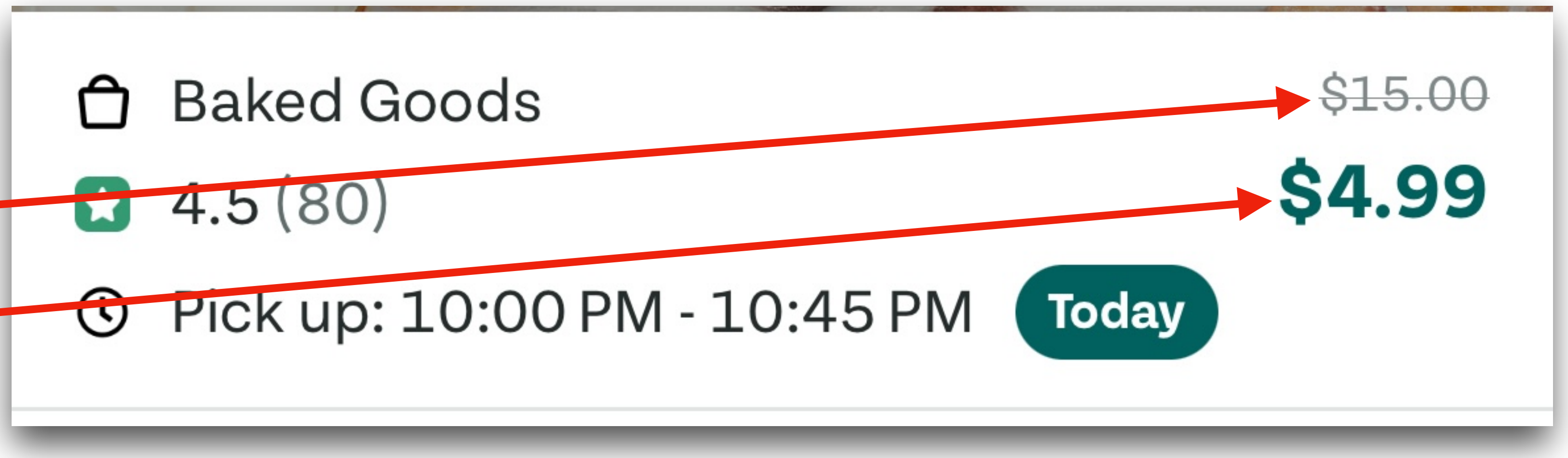

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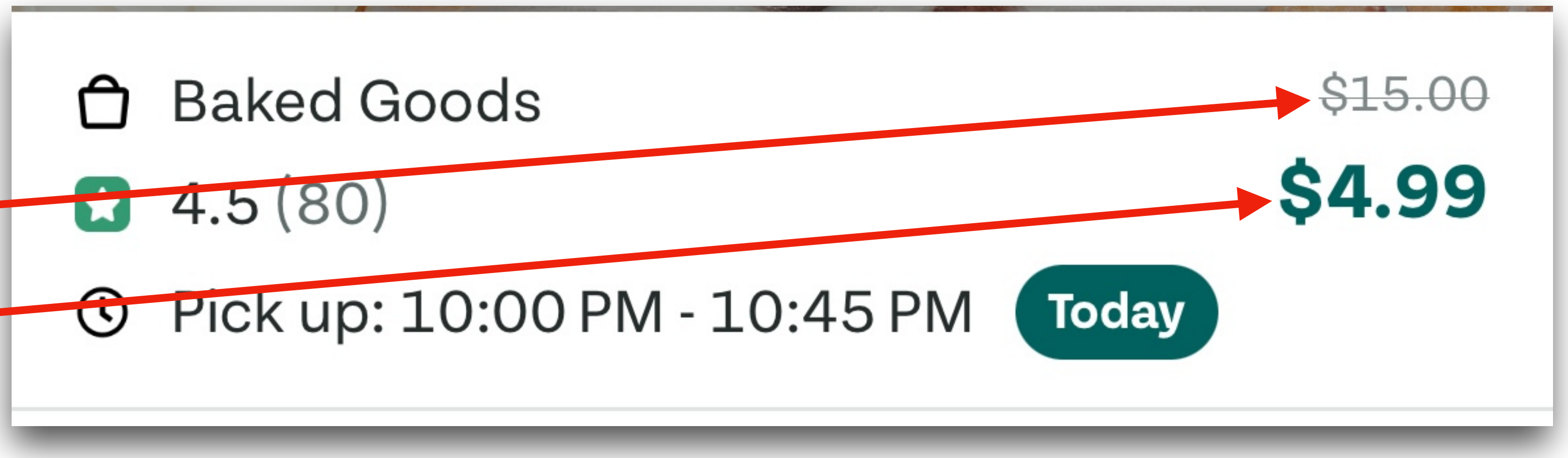

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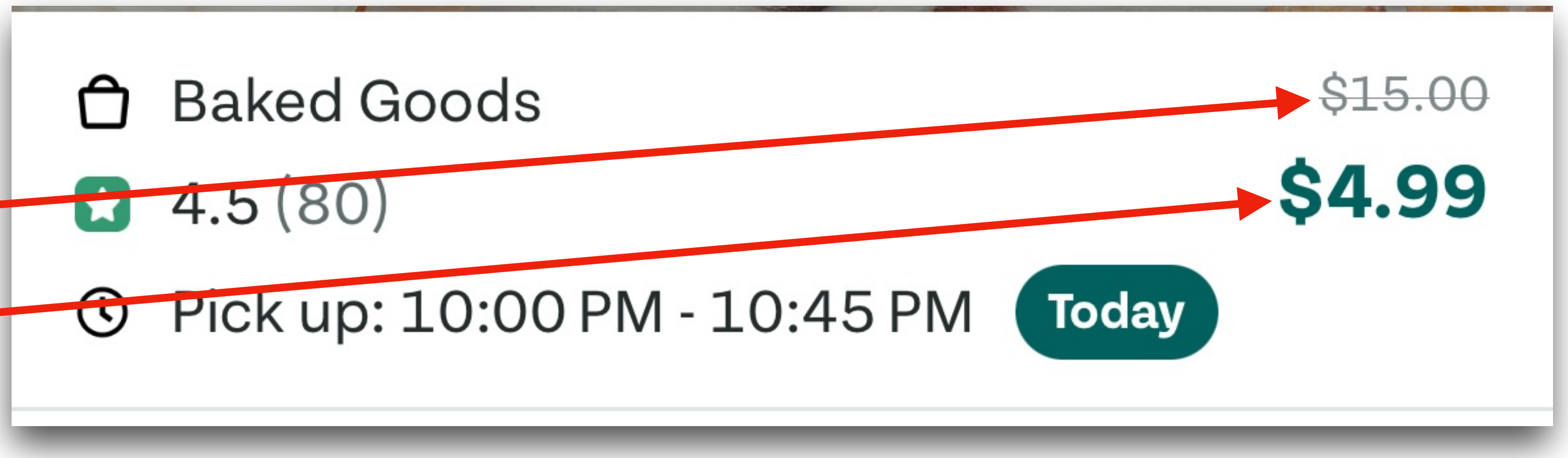

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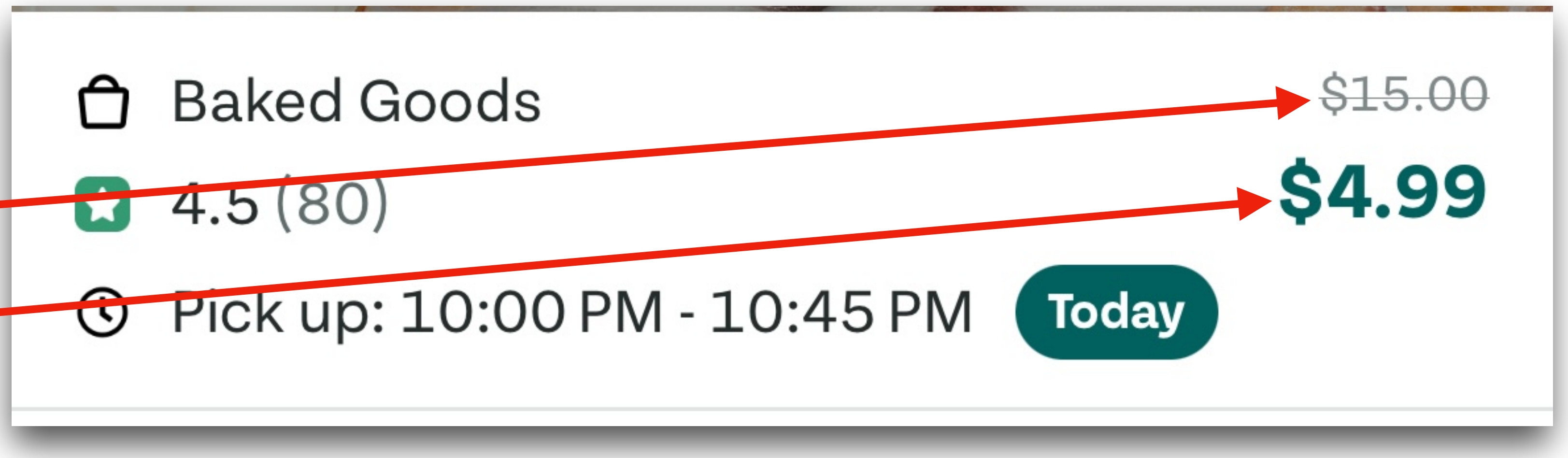

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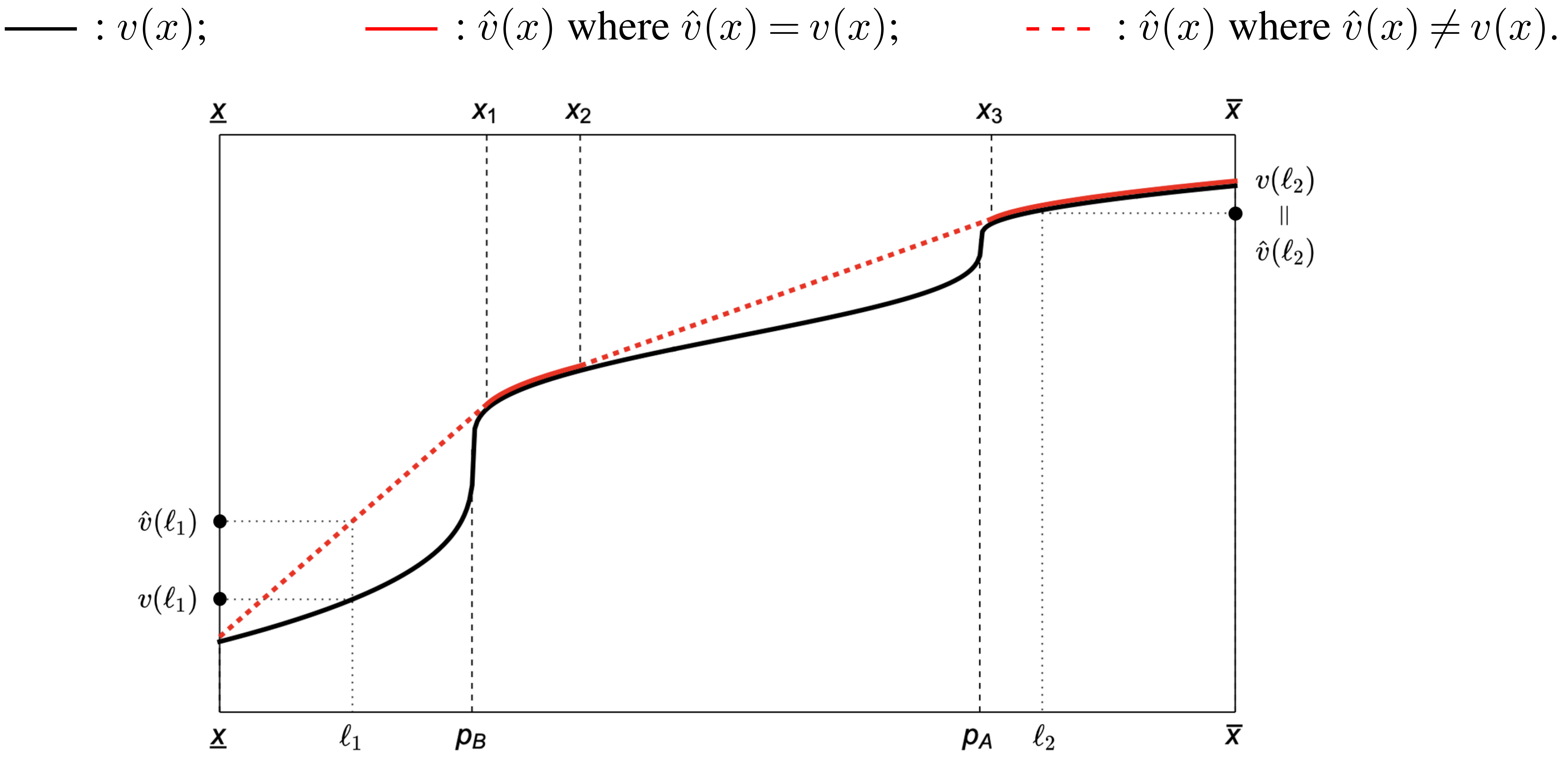
Consumer utility function $v(x)$ and its upper concave envelope $\hat{v}(x)$



Note. $\hat{v}(x) = v(x)$ for $x \in \{\underline{x}\} \cup [x_1, x_2] \cup [x_3, \bar{x}]$, and $\hat{v}(x) > v(x)$ for $x \in (\underline{x}, x_1) \cup (x_2, x_3)$. Note that $x_1 > p_B$ and $x_3 > p_A$.

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Proposition Under the 1-LA policy (i.e., myopic policy), the optimal bag number is $\tilde{n}^{(1)}(r) = D(r)$ and the optimal bag value is $\tilde{\ell}^{(1)}(r, Q) = \min\{\bar{x}, Q/D(r)\}$.

Performance Bound of Lookahead Approximation

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Proposition The revenue gap between the optimal policy and the K-LA policy is bounded by $J^*(r) - \tilde{J}^{(K)}(r) \leq \dfrac{\beta^K p\kappa\delta\zeta}{(1-(1-\delta)\beta)(1-\beta^K)}$.

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- **Remark:** the performance bound is tighter when:
  - Time discount factor β is small
  - Weight on new consumers' utility in reputation updating δ is small
  - Number of lookahead periods K is more
  - Maximal sensitivity of demand in response to reputation ζ is low
  - Maximal sensitivity of consumers' utility to food value κ is small

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Structure of Relaxed Policy

Structure of Relaxed Policy

Proposition $n^{MD}(r)$ increases in r and $\ell^{MD}(r)$ decreases in r. Specifically, if $p \geq c$, then $n^{MD}(r) = D(r)$.

Otherwise, if $p < c$, we have $n^{MD}(r) = D(r) - \left[\ell^{MD}(r)D(r) - \bar{Q}\right]^+$, where $\ell^{MD}(r)$ is the solution to $\max_{\ell \in [0,1]} \; p\bar{Q} + (1-\ell)\bar{n}p + \beta J^{MD}\left[\delta \hat{v}(\ell) + (1-\delta)r\right]$.

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- When $p \geq c$
  - The store maximizes short-term profits by distributing the maximum number of bags, adjusting the average bag value based on leftovers
- When $p < c$
  - The store reduces the number of bags to optimize costs, using the available leftover food more efficiently

Performance Bound of Relaxed Policy

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Proposition It holds that $J^{\pi_D} \leq J^*(r) \leq J^M(r) \leq J^{MD}(r)$, and

$$J^*(r) - J^{\pi_D}(r) \leq J^{MD}(r) - J^{\pi_D}(r) \leq \frac{c}{1-\beta} \cdot \left[\frac{\sigma}{2} + \left(\sqrt{D(r)} - \sqrt{\bar{Q}/\bar{x}}\right)^2\right]$$

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- **Remark:** the performance bound is tighter when
  - Smaller time discount factor β
  - Lower supplementary costs c
  - Smaller standard deviation σ
  - Aligning $\bar{Q}$ with $xD(r)$

Steady State Analysis

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Proposition Under the optimal policy with deterministic leftover, the steady-state bag value converges to ℓ^*, corresponding to the store's steady-state reputation $r^* = \hat{v}(\ell^*)$ and bag number $n^* = D(r^*) = D[\hat{v}(\ell^*)]$. The steady-state bag value is the unique solution to

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- **Implication:** Stores do not need to maintain a *perfect* rating
- Each store reaches a different long-term reputation level, with key influencing factors:
  - Cost structure
  - Consumer preferences
  - Demand dynamics
  - Reputation update mechanism

Numerical Experiments

Numerical Experiments

- Experiment setup
  - We assume a simple logstic utility function with one reference point, given by
    $$v(x) = \frac{1}{1 + e^{-10(x-0.5)}}.$$
  - Its upper concave envelope can be expressed as
    $$\hat{v}(x) = \begin{cases} 1.25x & \text{if } x \in [0, 0.676]; \\ \frac{1}{1 + e^{-10(x-0.5)}} & \text{if } x \in (0.676, 1]. \end{cases}$$

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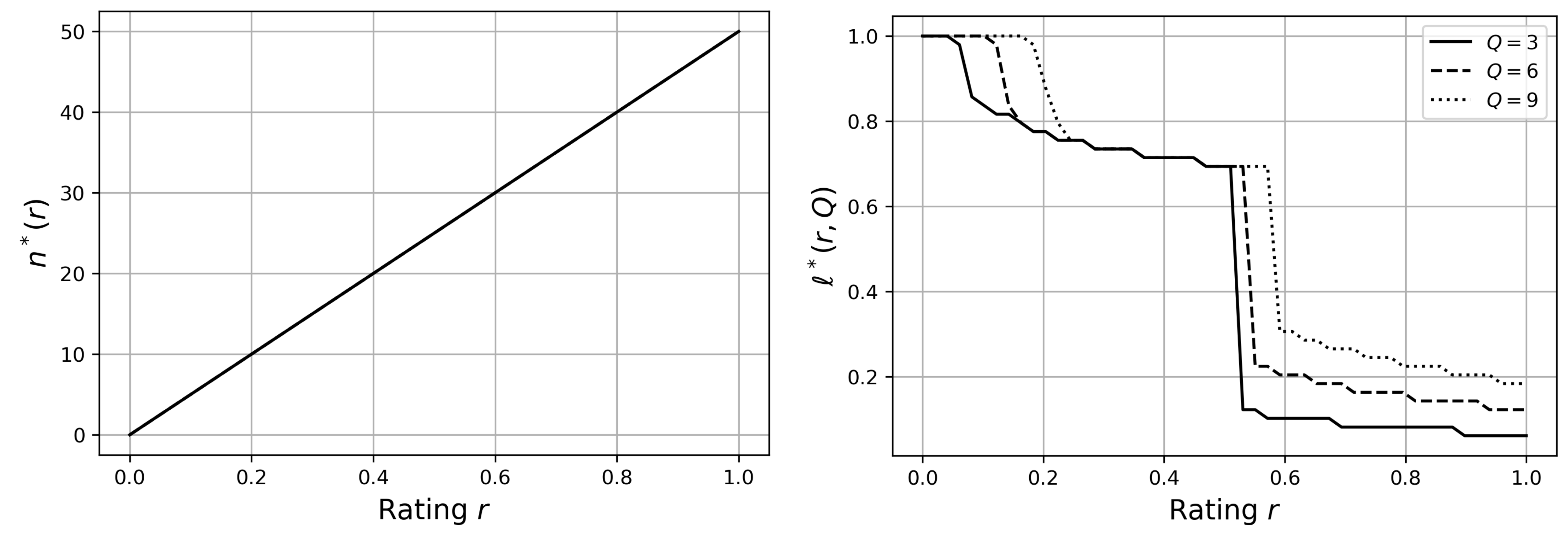
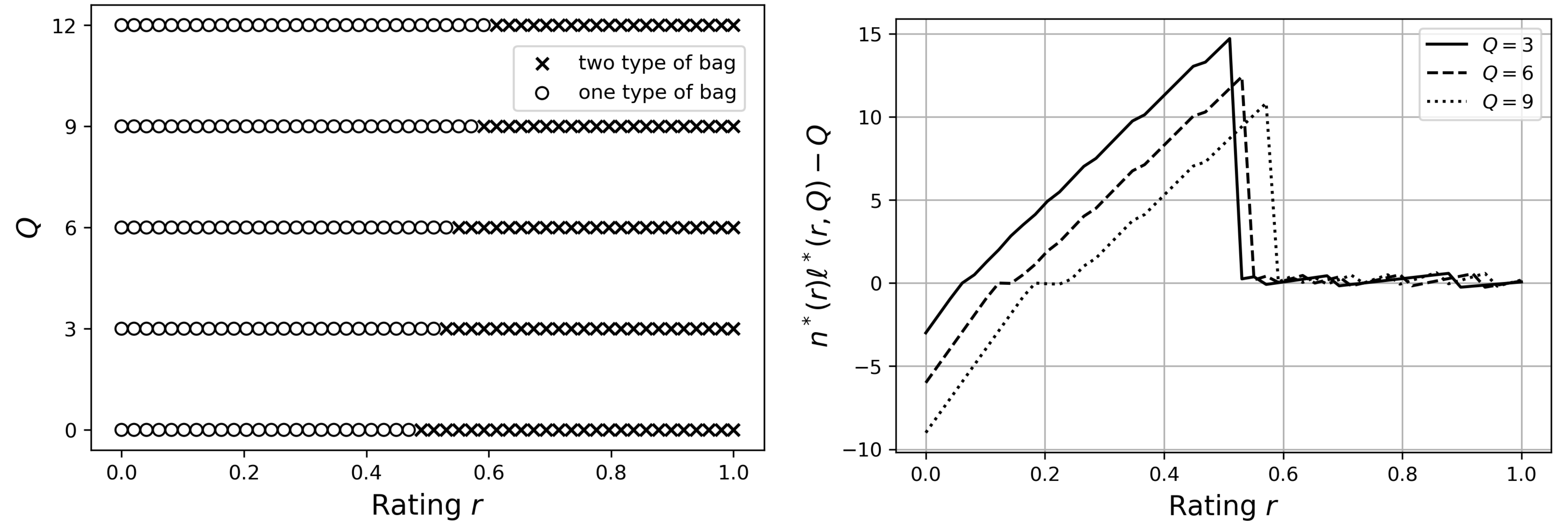
Optimal policy



(a) Optimal bag number $n^*(r)$ (b) Optimal average bag value $\ell^*(r, Q)$



(c) Number of support points of $\phi^*(\cdot \mid \ell^*(r, Q))$ (d) Supplementary at optimality $n^*(r)\ell^*(r, Q) - Q$

Policy Comparison: Revenue

- We compare four policies: i) the optimal policy π^*, ii) the 2-LA policy $\pi^{(2)}$, iii) the deterministic approximation policy π_D, and iv) the naive policy π_N.

Average revenues under different policies

Parameters $(c, \delta), Q \sim F$	Optimal	2-LA	DFA	Naive
$(5, 0.3), Q \sim U[0, 12]$	844.12 ± 13.91	841.82 ± 14.39	793.92 ± 10.44	787.78 ± 25.27
$(5, 0.6), Q \sim U[0, 12]$	931.70 ± 14.03	921.89 ± 14.07	870.41 ± 9.04	867.68 ± 27.23
$(10, 0.3), Q \sim U[0, 12]$	827.59 ± 21.78	826.44 ± 21.89	796.61 ± 23.44	794.27 ± 32.37
$(10, 0.6), Q \sim U[0, 12]$	910.14 ± 18.10	900.15 ± 18.10	858.10 ± 19.05	880.94 ± 25.72
$(5, 0.3), Q \sim U[3, 9]$	850.48 ± 10.77	849.39 ± 11.20	829.41 ± 7.32	849.96 ± 11.43
$(5, 0.6), Q \sim U[3, 9]$	921.94 ± 7.18	891.93 ± 7.20	916.91 ± 5.87	890.39 ± 7.64
$(10, 0.3), Q \sim U[3, 9]$	856.73 ± 8.80	855.63 ± 8.76	801.22 ± 10.99	859.46 ± 8.78
$(10, 0.6), Q \sim U[3, 9]$	936.56 ± 9.60	926.56 ± 9.60	889.64 ± 13.54	909.09 ± 9.61

Note: $\pm$ indicates the half-width of the 95% confidence interval for the estimated means of each metric.

Policy Comparison: Waste

- We compare four policies: i) the optimal policy π^*, ii) the 2-LA policy $\pi^{(2)}$, iii) the deterministic approximation policy π_D, and iv) the naive policy π_N.

Average in-store waste under different policies

Parameters $(c, \delta), Q \sim F$	Optimal	2-LA	DFA	Naive
$(5, 0.3), Q \sim U[0, 12]$	6.44 ± 2.07	6.59 ± 2.08	22.82 ± 2.72	8.15 ± 1.66
$(5, 0.6), Q \sim U[0, 12]$	5.17 ± 2.14	5.19 ± 2.14	22.12 ± 2.27	8.20 ± 1.66
$(10, 0.3), Q \sim U[0, 12]$	7.86 ± 2.03	8.09 ± 2.01	24.36 ± 3.20	9.06 ± 1.60
$(10, 0.6), Q \sim U[0, 12]$	6.83 ± 1.76	6.83 ± 1.76	22.74 ± 3.23	8.56 ± 1.26
$(5, 0.3), Q \sim U[3, 9]$	6.78 ± 1.19	6.94 ± 1.19	14.75 ± 1.74	6.77 ± 1.19
$(5, 0.6), Q \sim U[3, 9]$	6.65 ± 1.17	6.65 ± 1.17	13.99 ± 1.26	6.62 ± 1.17
$(10, 0.3), Q \sim U[3, 9]$	7.13 ± 0.90	7.60 ± 0.93	15.94 ± 1.54	6.77 ± 0.90
$(10, 0.6), Q \sim U[3, 9]$	6.26 ± 1.30	6.26 ± 1.30	15.85 ± 1.41	5.87 ± 1.33

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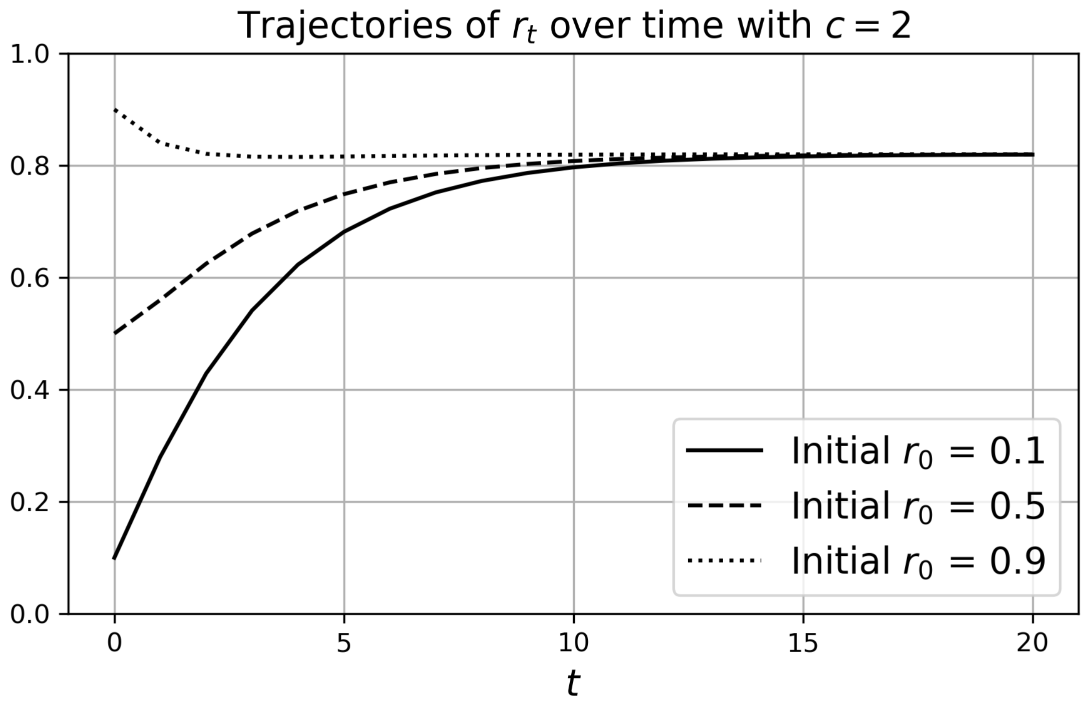
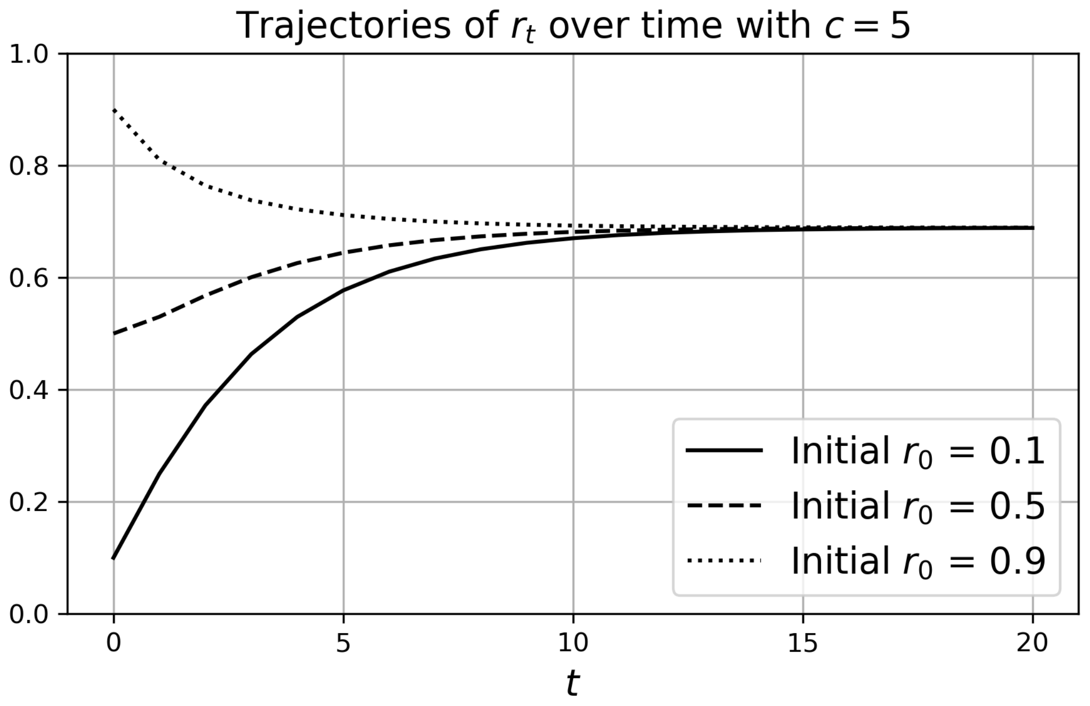
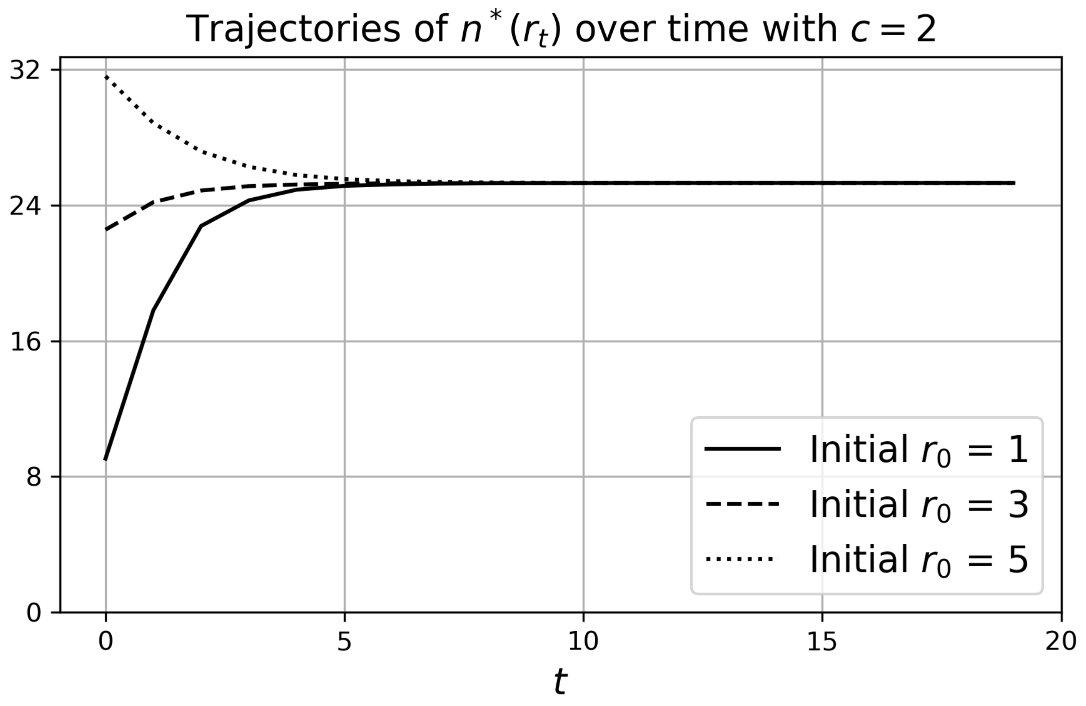
Numerical Experiments: System Convergence

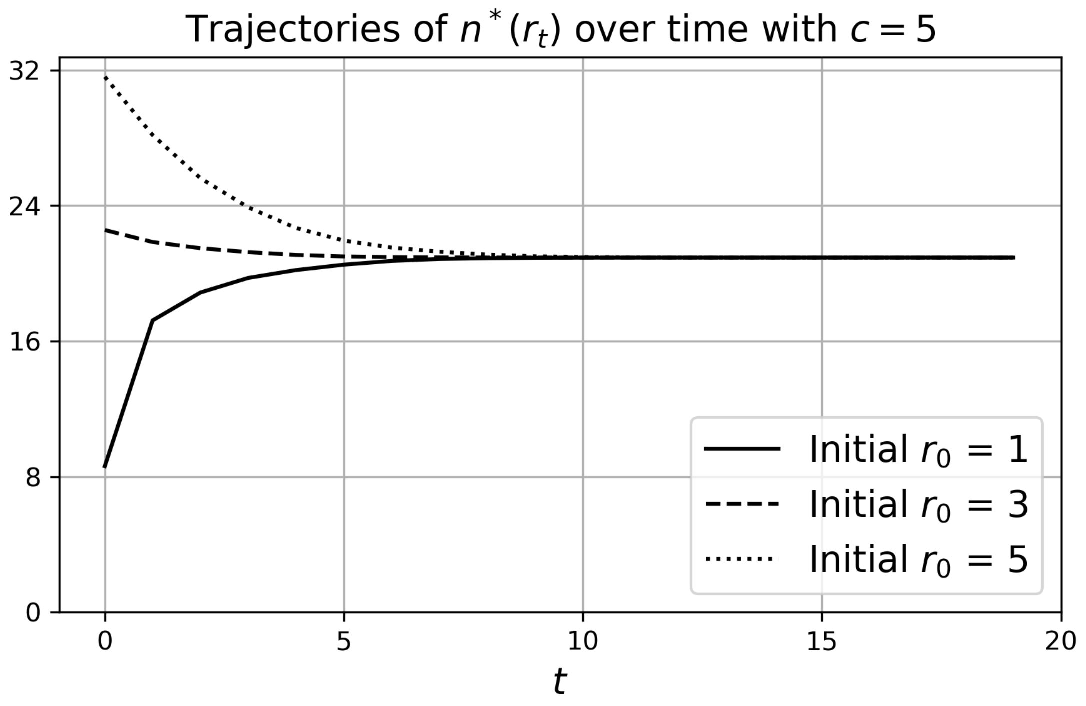
Reputation trajectory



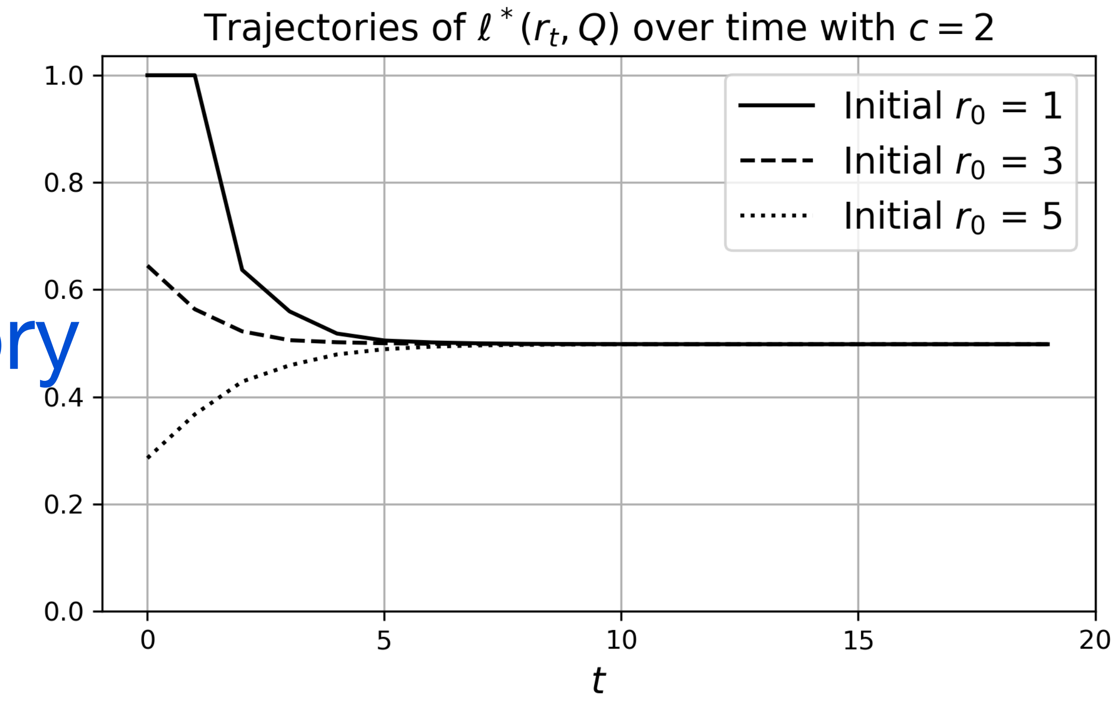


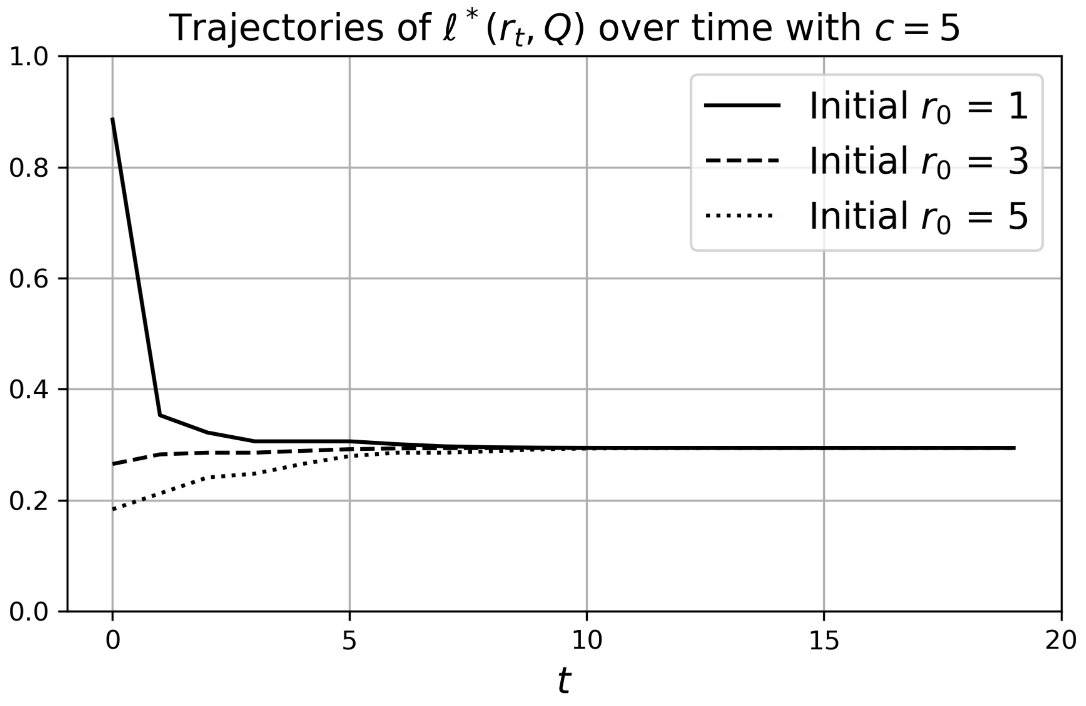
Bag number trajectory





Average bag value trajectory





Left: Low c/p ratio

Right: High c/p ratio

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Future Directions

- End-to-end management/ Information disclosure / Pricing effects /…

Thank you for your attention!

Questions and comments are appreciated!
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