Intertemporal Pricing in the Presence of Consumer Behaviors

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YinzOR 2023
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Junyu Cao, UT Austin

Amy Guo, UC Berkeley

Max Shen, HKU

“Intertemporal Pricing via Nonparametric Estimation: Integrating Reference Effects and Consumer Heterogeneity”. *Manufacturing & Service Operations Management.* **H. Jiang**, Junyu Cao, Z.-J. Max Shen.

“Multi-Product Dynamic Pricing with Reference Effects Under Logit Demand”. Under 2nd-round review at *Operations Research.* Amy Guo, **H. Jiang**, Z.-J. Max Shen.

Pricing for a Large Market

JD.COM

China's leading e-commerce platform

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90%+
Orders Placed Through Mobile

588.3m
Annual Active Customers

1,500+
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2021 Net Revenues

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- Pricing strategies to boost revenue

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- Data from thousands of consumers for one product
- Exact timestamps of individual consumer activities

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Snapshots of a cereal product from Amazon's website

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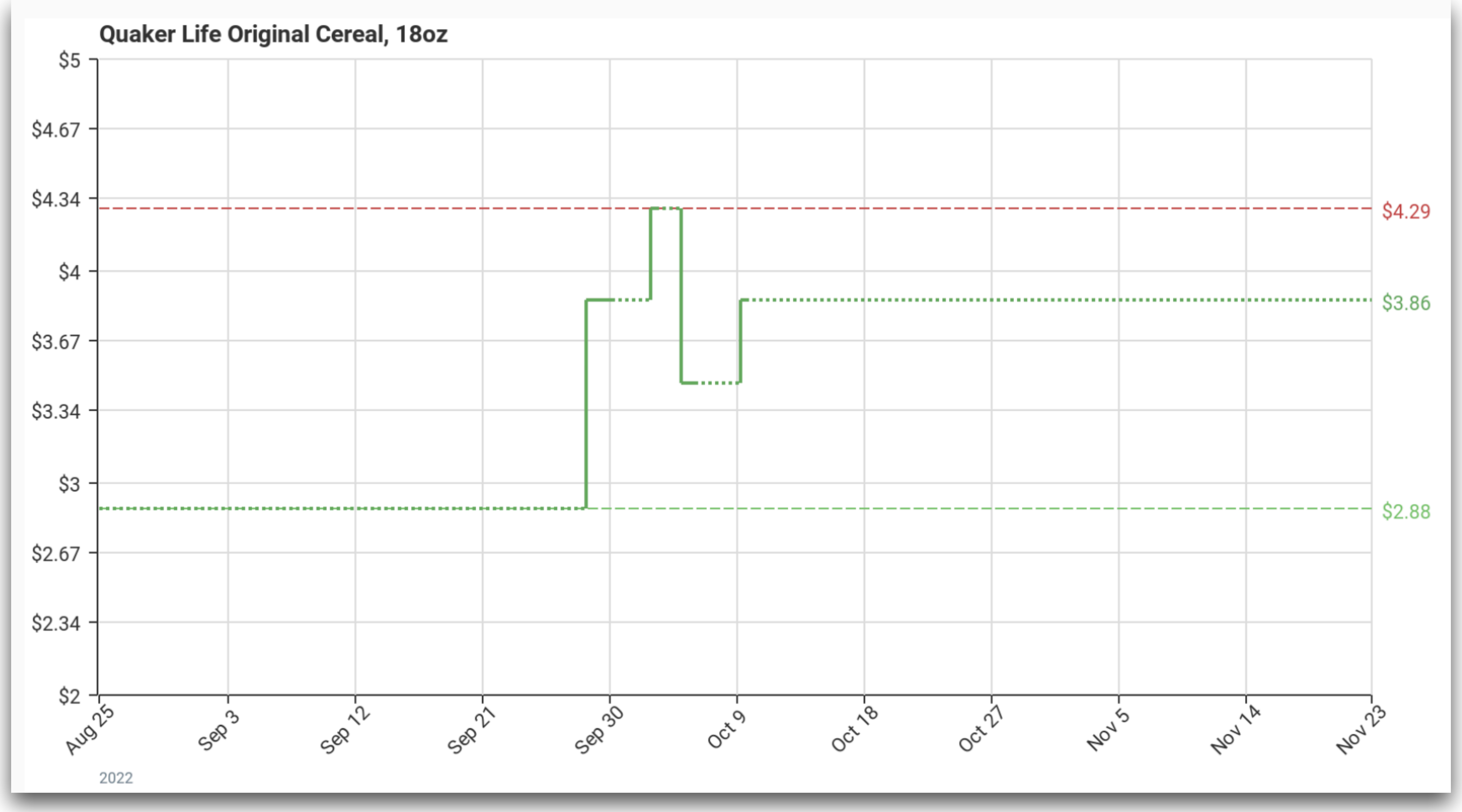

Historical prices of this cereal product (source: camelcamelcamel.com)

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Homogeneous consumer

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Aggregate market data

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Known deterministic demand

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Optimal price is a fixed point

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Theory	Practice
Homogeneous consumer	
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Balancing Theory and Practice

Theory	Practice
Homogeneous consumer	Heterogeneous consumer
Aggregate market data	
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Balancing Theory and Practice

Theory	Practice
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Tractability

Balancing Theory and Practice

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Tractability — Practicality

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Intertemporal Aspects of Pricing

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How do consumer valuations depend on historical prices?

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Marketing: Empirics

Reference prices are shaped by historical prices

Intertemporal Aspects of Pricing

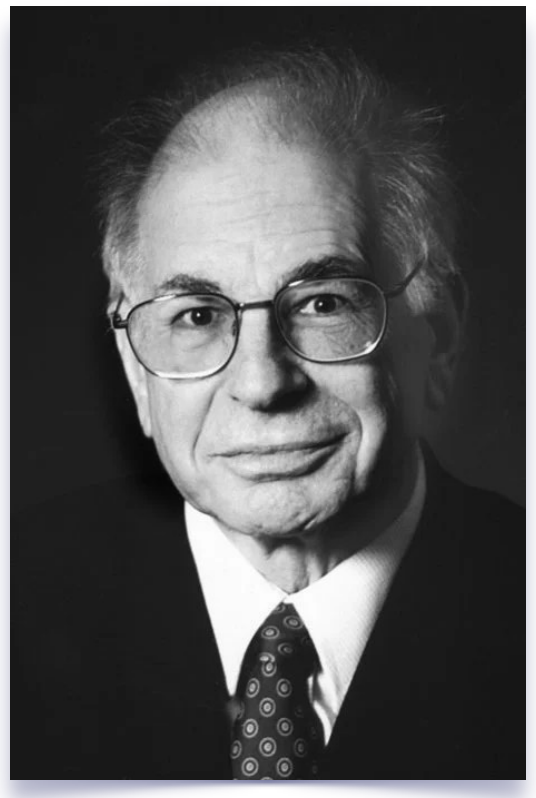
How do consumer valuations depend on historical prices?

Marketing: Empirics

Reference prices are shaped by historical prices

Economics: Prospect theory

Reference prices affect consumer valuations in an asymmetric way



Daniel Kahneman (Nobel Prize in Economics, 2002)

Heterogeneous Consumer Model

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Time t

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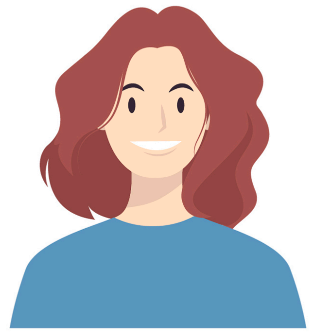

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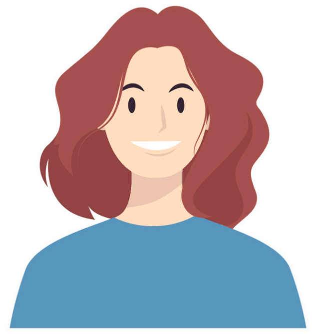
Time t



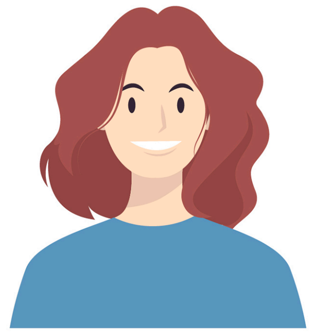
p_t

Heterogeneous Consumer Model

Before t

$p_1, p_2, \ldots, p_{t-1}$

Time t



p_t

Heterogeneous Consumer Model

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r_t

p_t

Heterogeneous Consumer Model

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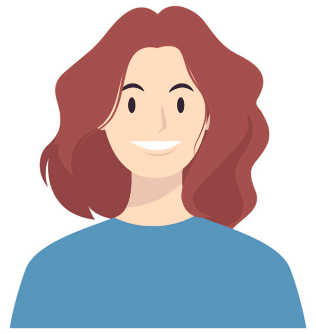
Time t



r_t

p_t

Utility $u_t = a - bp_t + c_+(r_t - p_t)_+ + c_-(r_t - p_t)_- + \epsilon_t$

Heterogeneous Consumer Model

Before t

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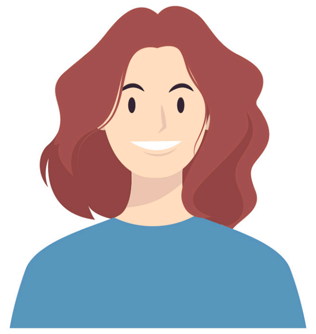

r_t

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Price sensitivity

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r_t

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Gain: $\max(r_t - p_t, 0)$

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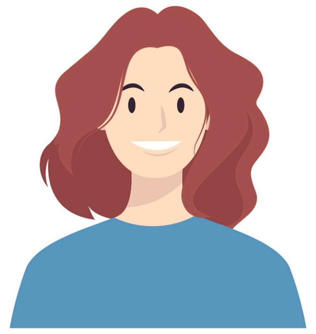

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Sensitivity to gain

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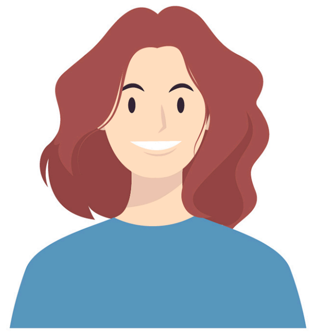

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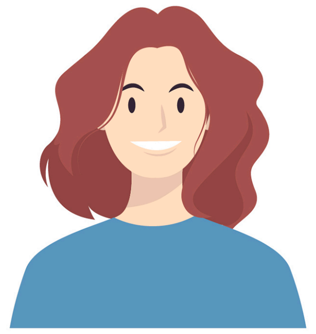

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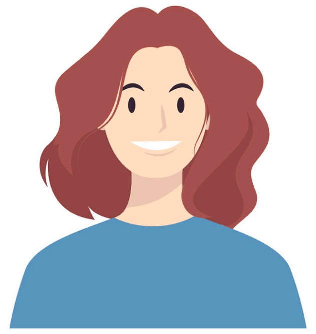

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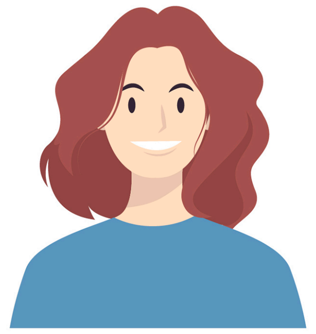

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Price sensitivity; Sensitivity to gain; Sensitivity to loss; Random iid Gumbel error

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Heterogeneous consumers $\boldsymbol{\theta} \sim G$

Key: No parametric assumption is imposed on G!

$$\mathrm{P}^G(r_t, p_t) = \int_{\boldsymbol{\theta}\in\Theta} \frac{\exp\{u_t(\boldsymbol{\theta})\}}{\exp\{u_t(\boldsymbol{\theta})\} + 1} \mathrm{d}G(\boldsymbol{\theta})$$

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"Diminishing sensitivity" property

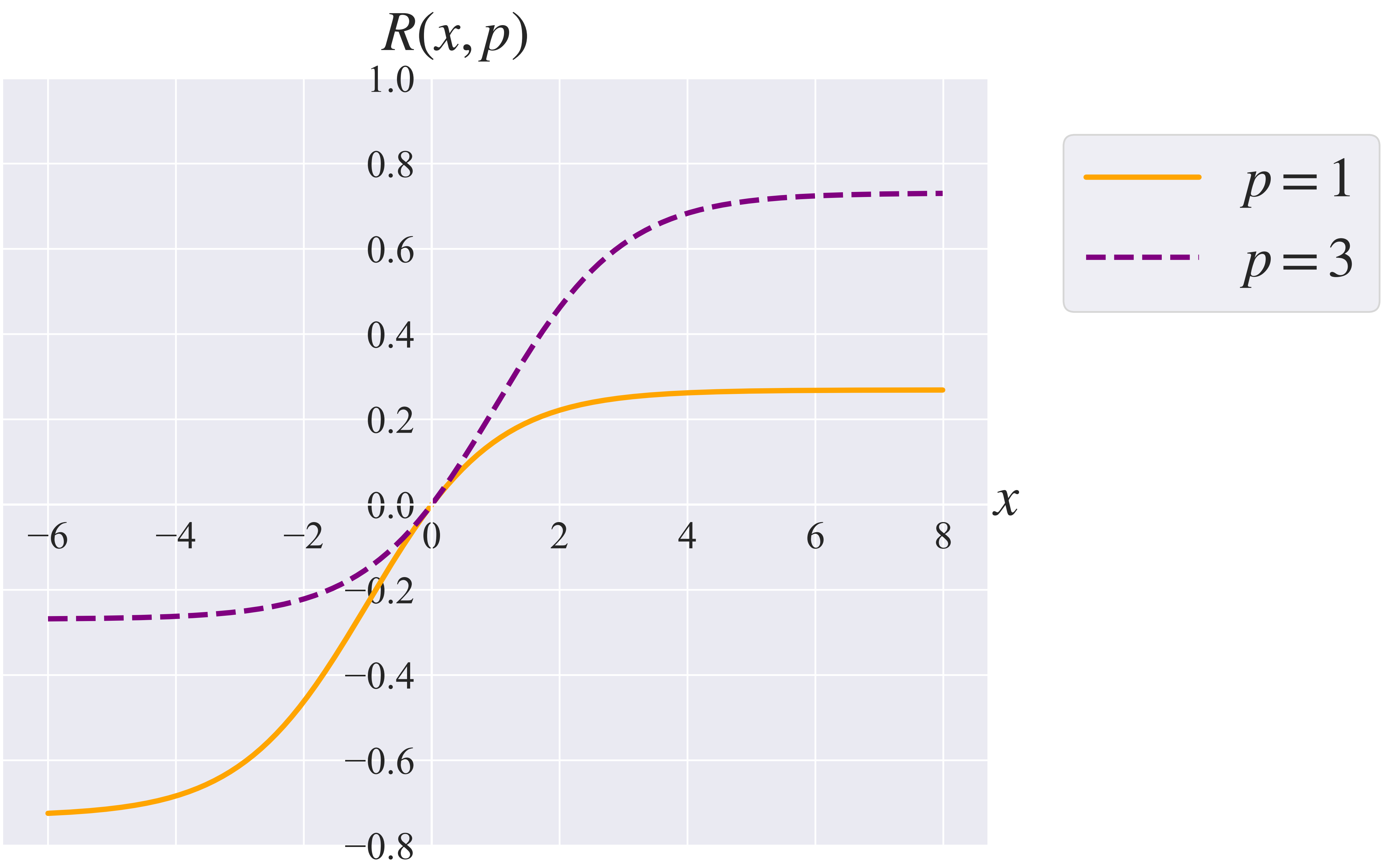

Illustration of reference effects ($a = 2, b = 1, c_+ = c_- = 1$)

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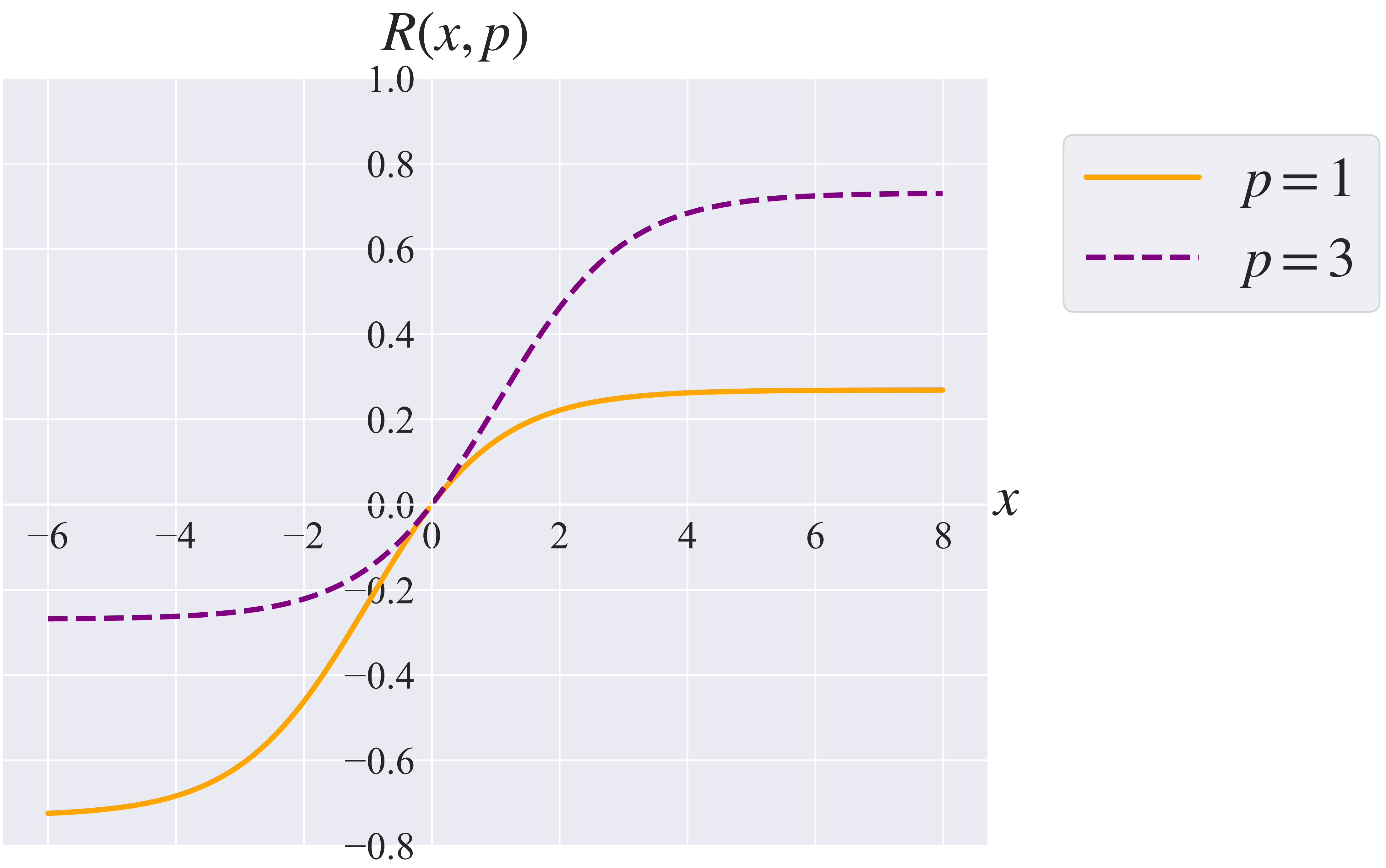

Illustration of reference effects ($a = 2, b = 1, c_+ = c_- = 1$)

"The first sip of a drink tastes the best,

and the first dollar lost hurts the most."

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Goal

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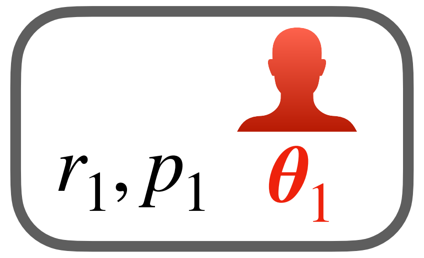

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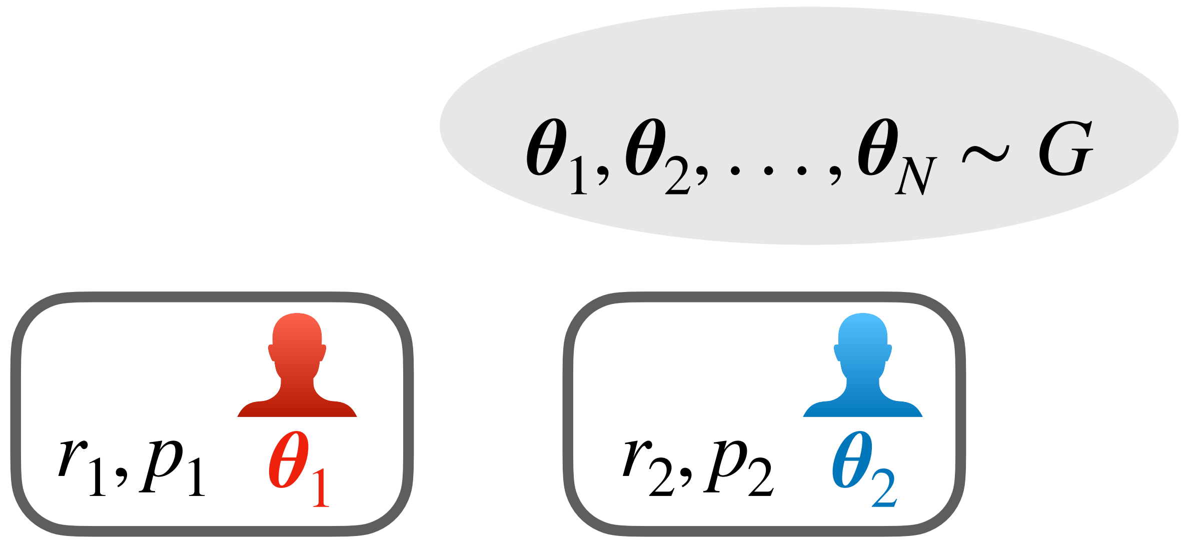

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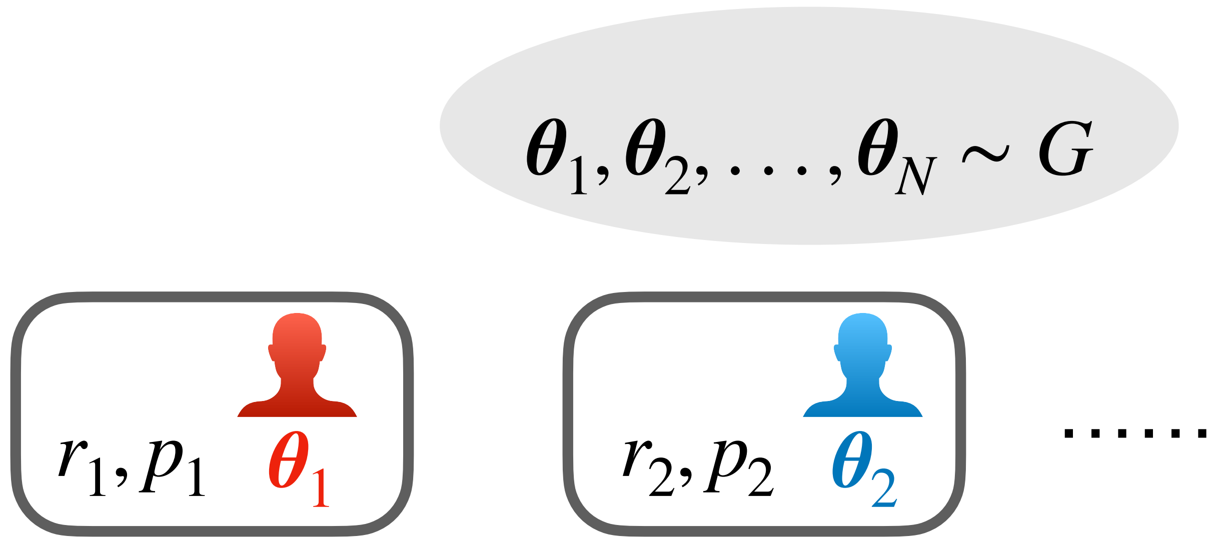

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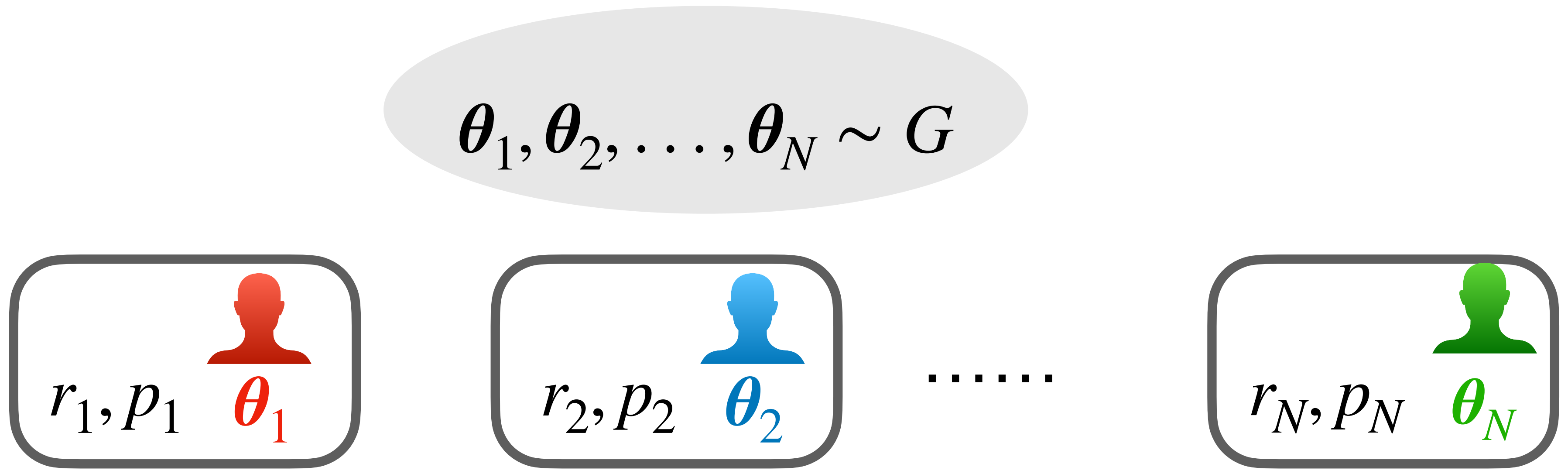

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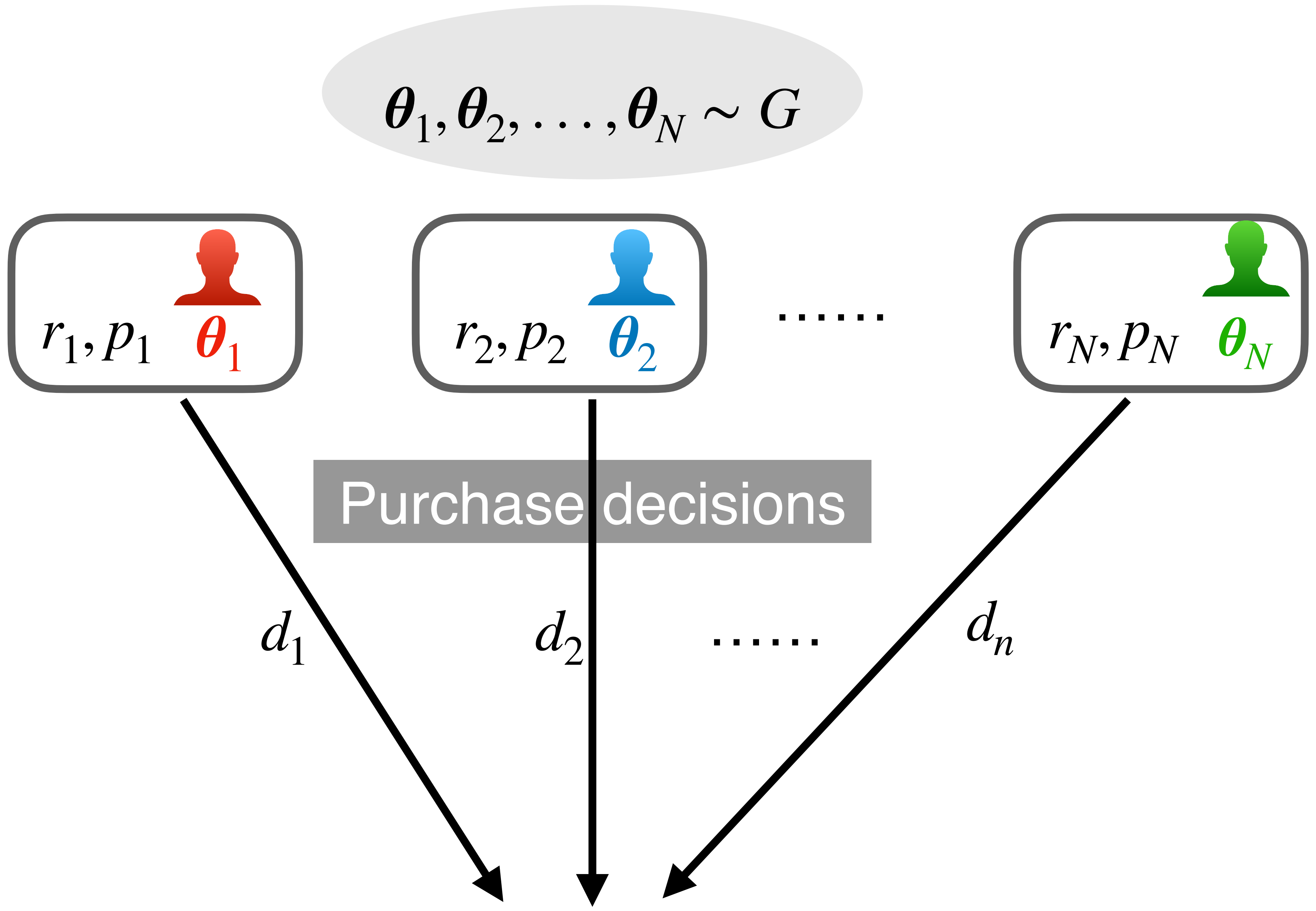

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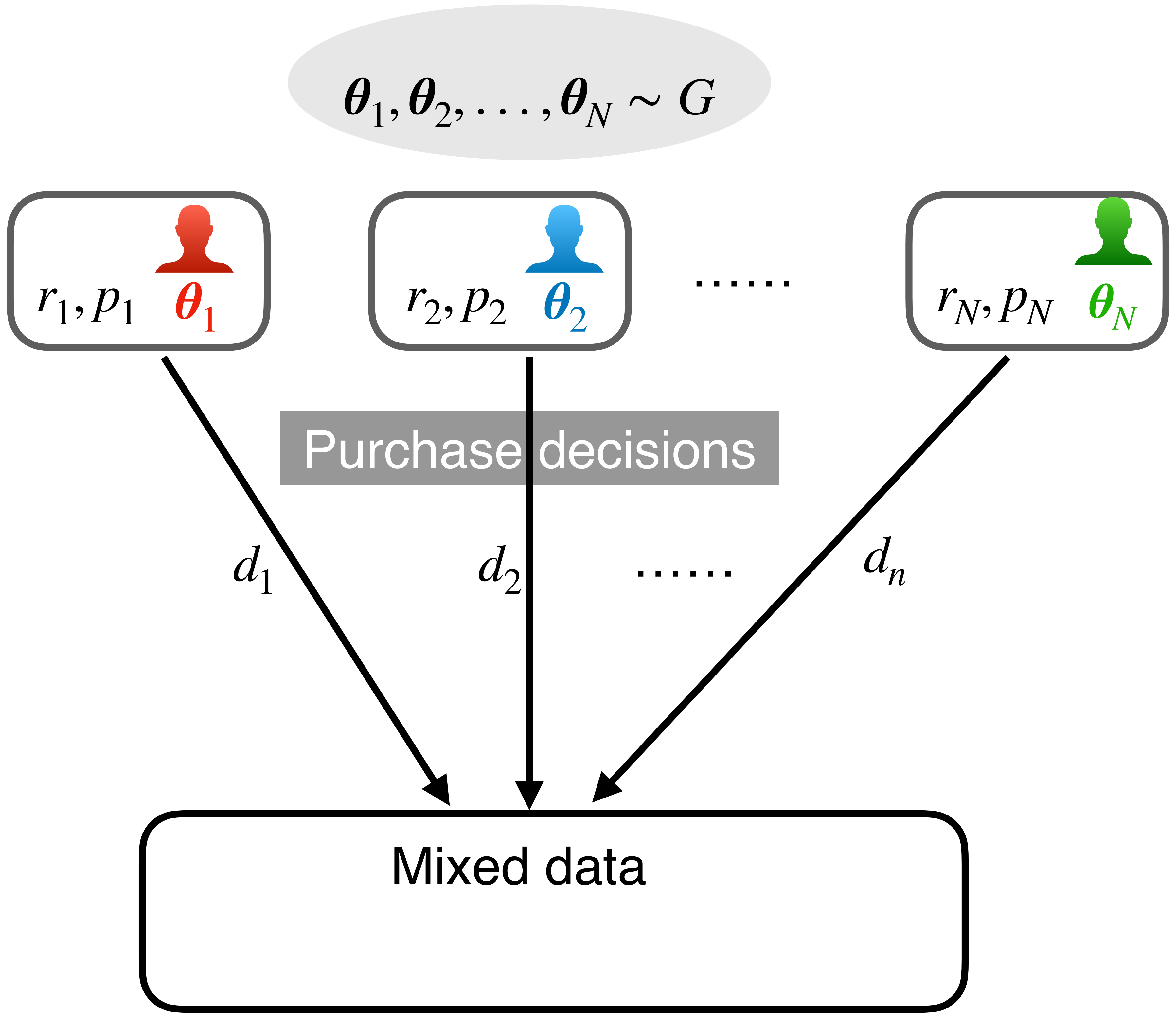

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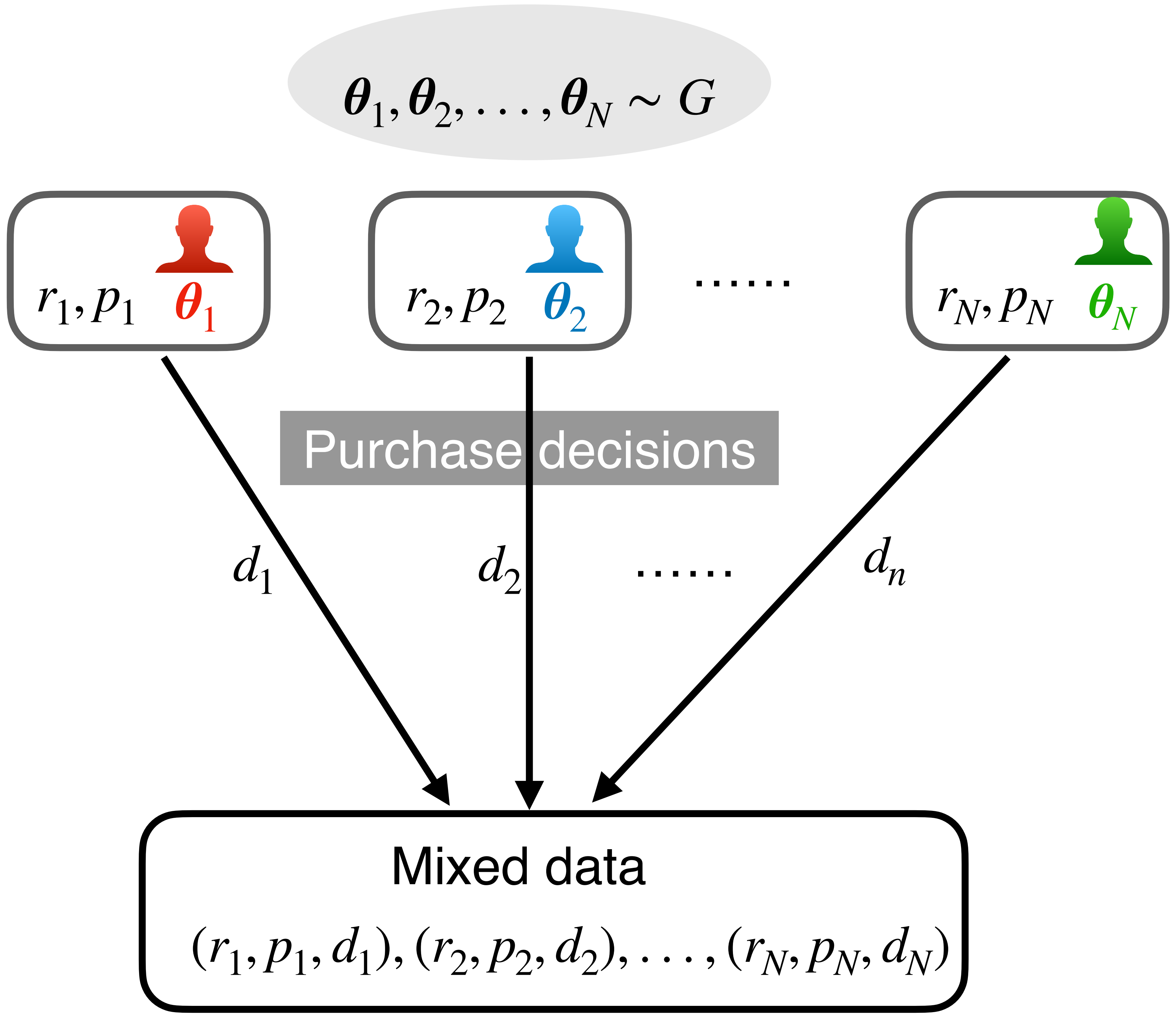

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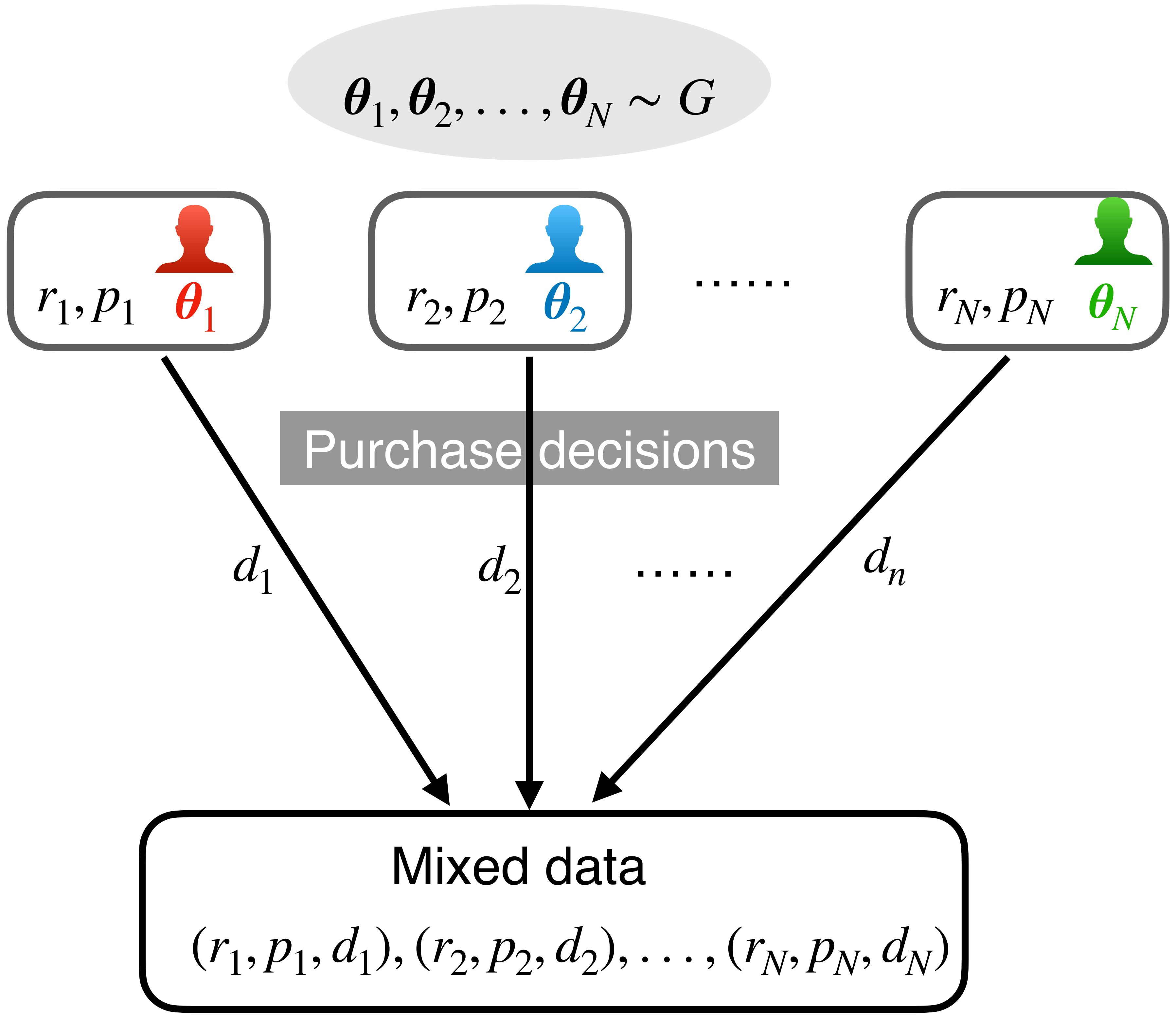

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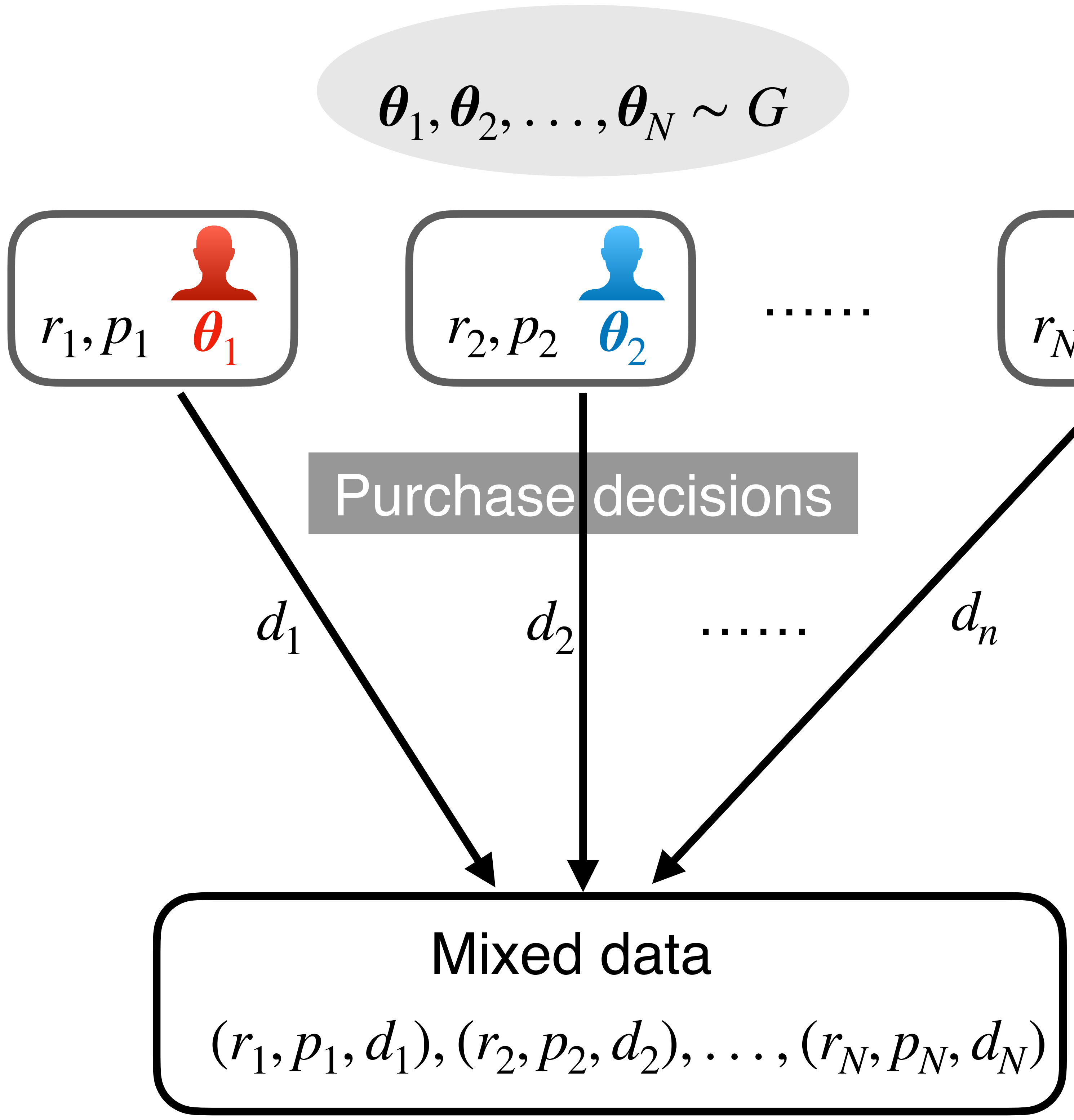

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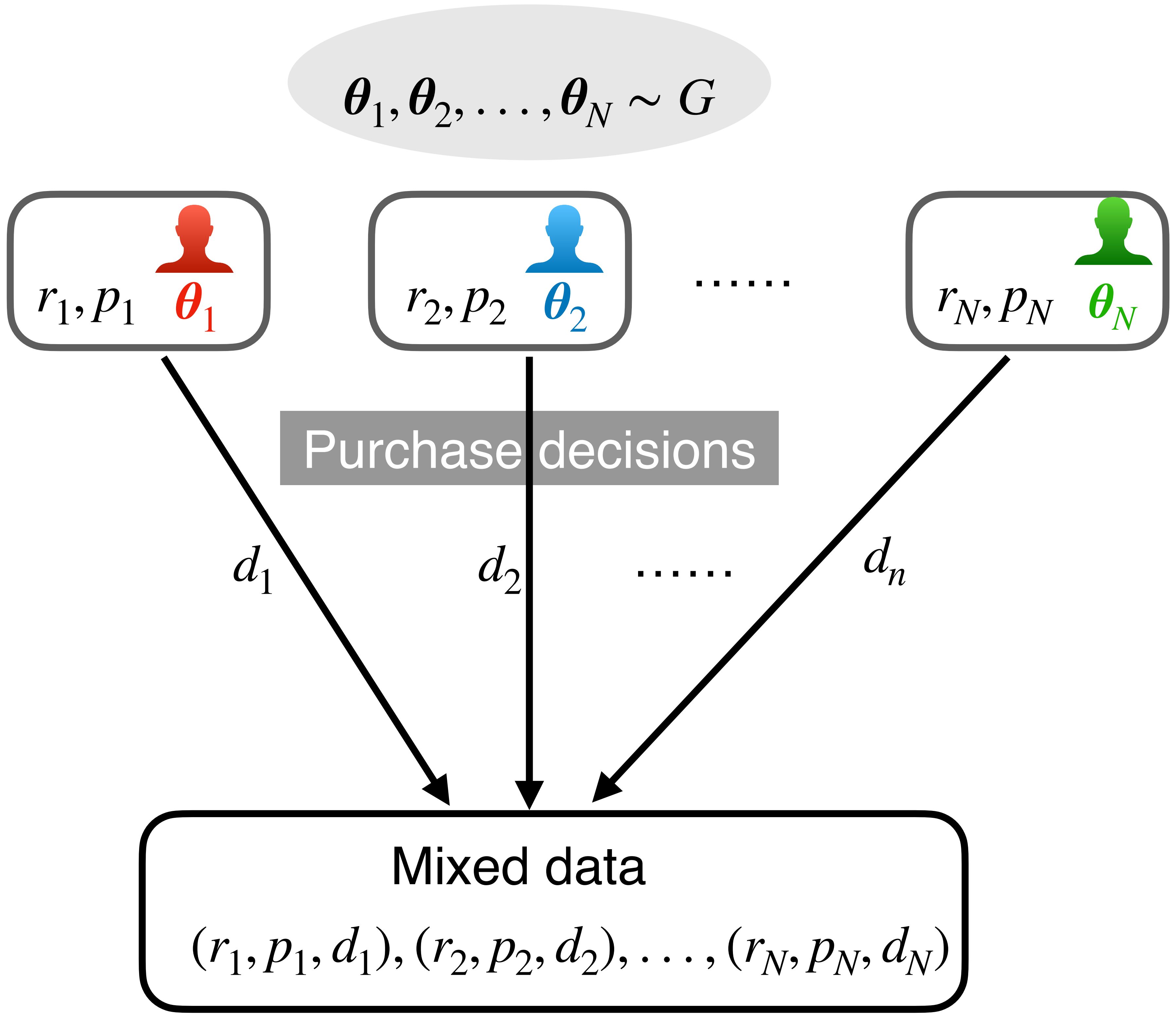

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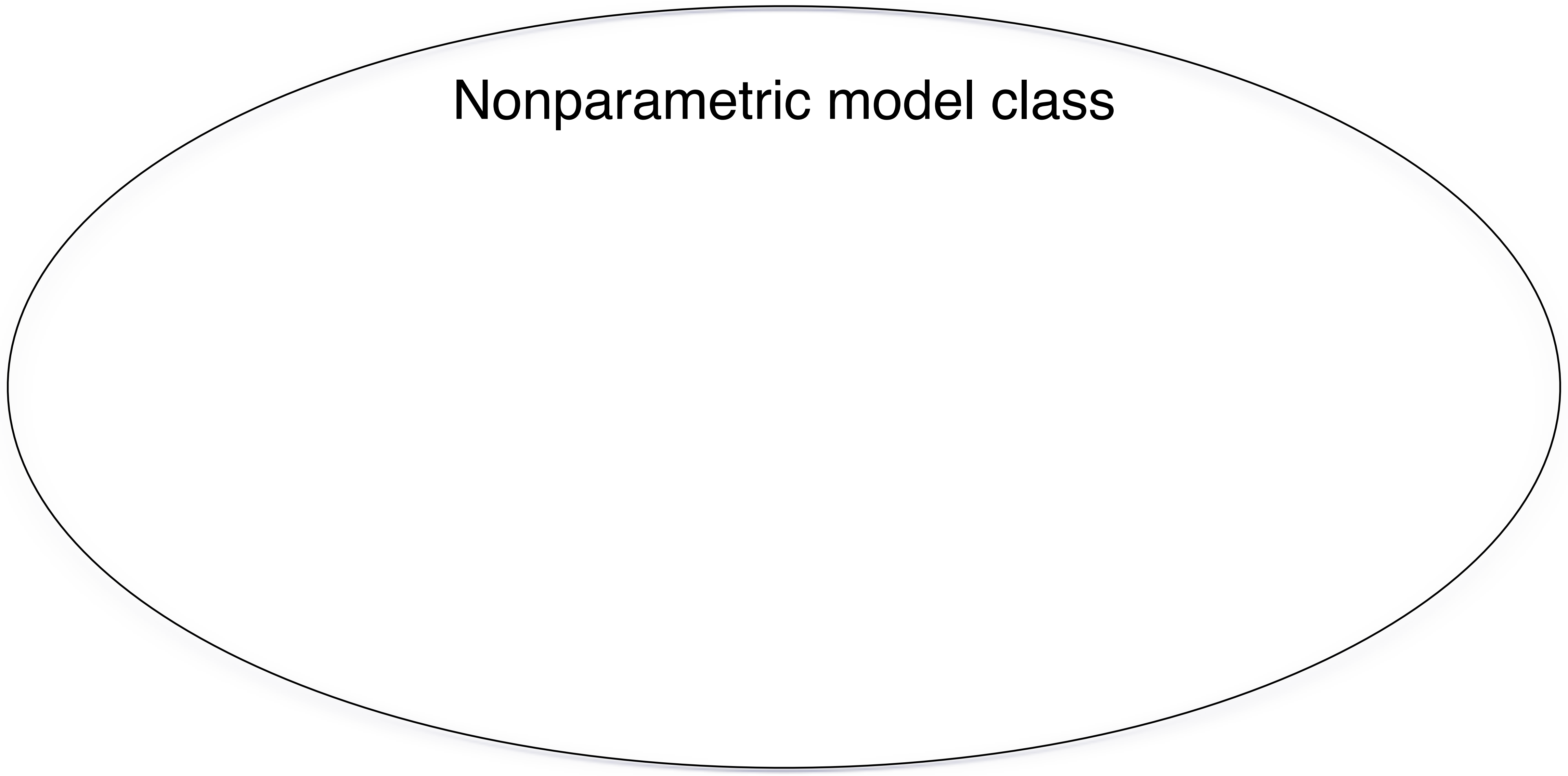

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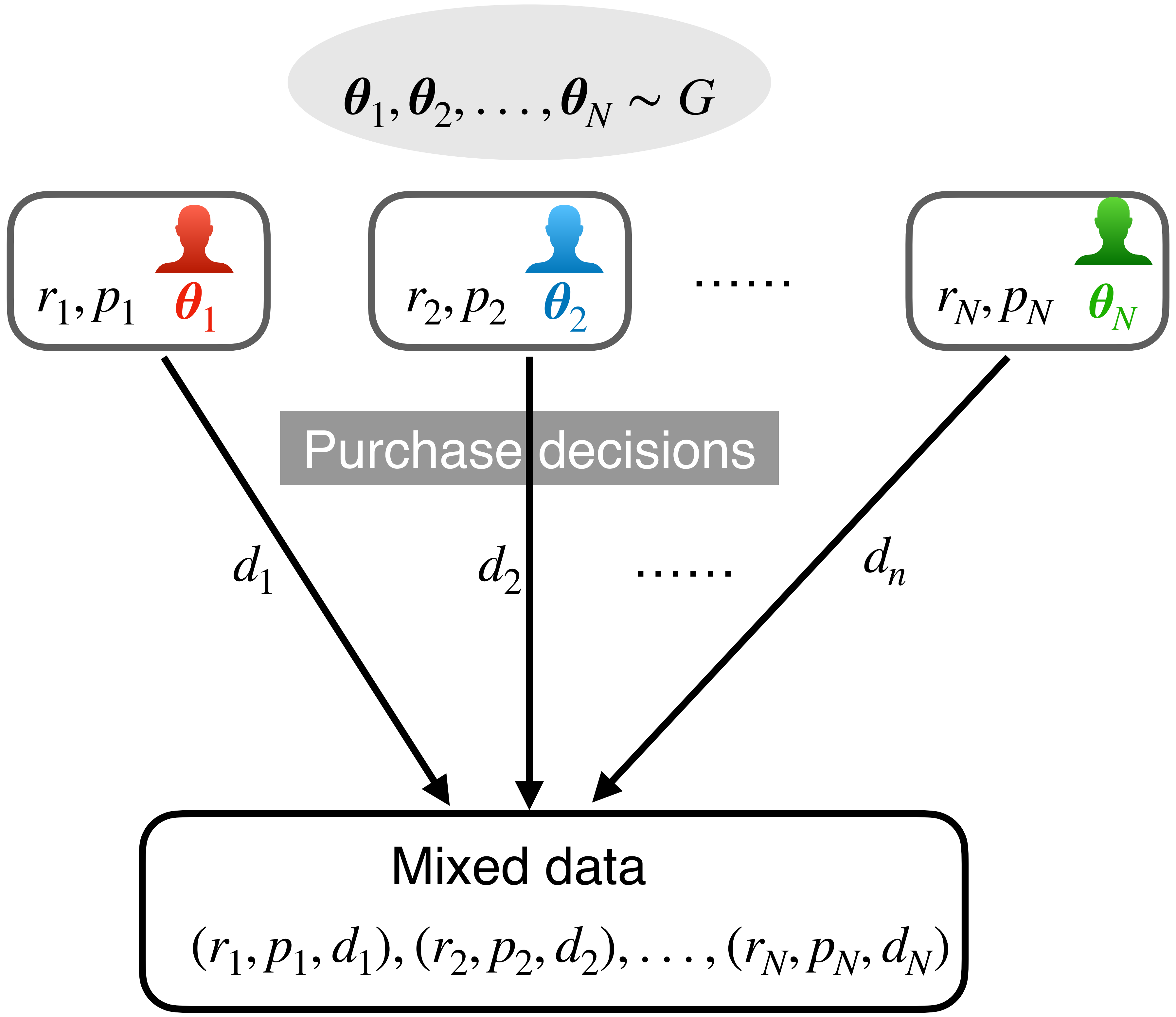

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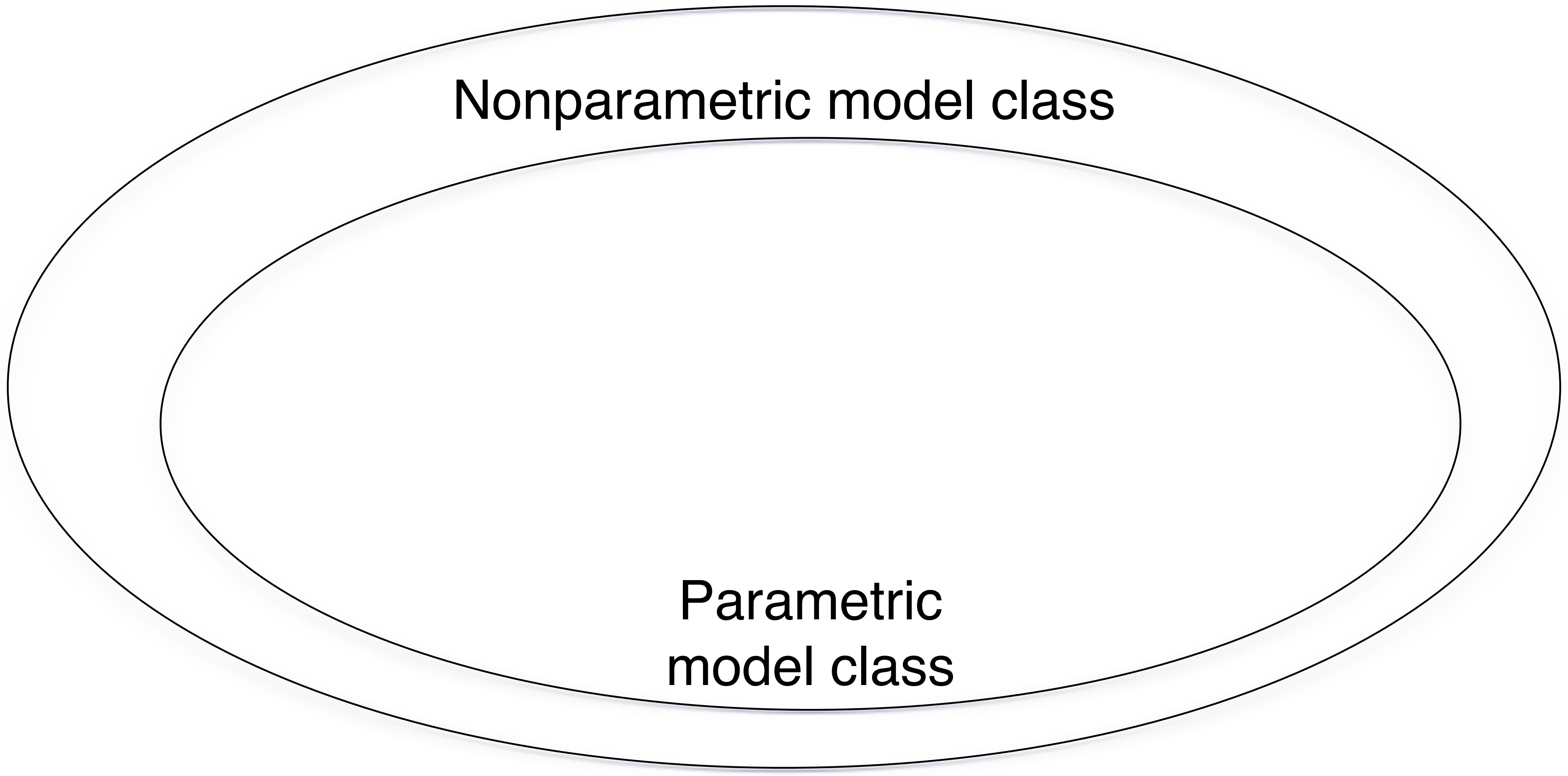

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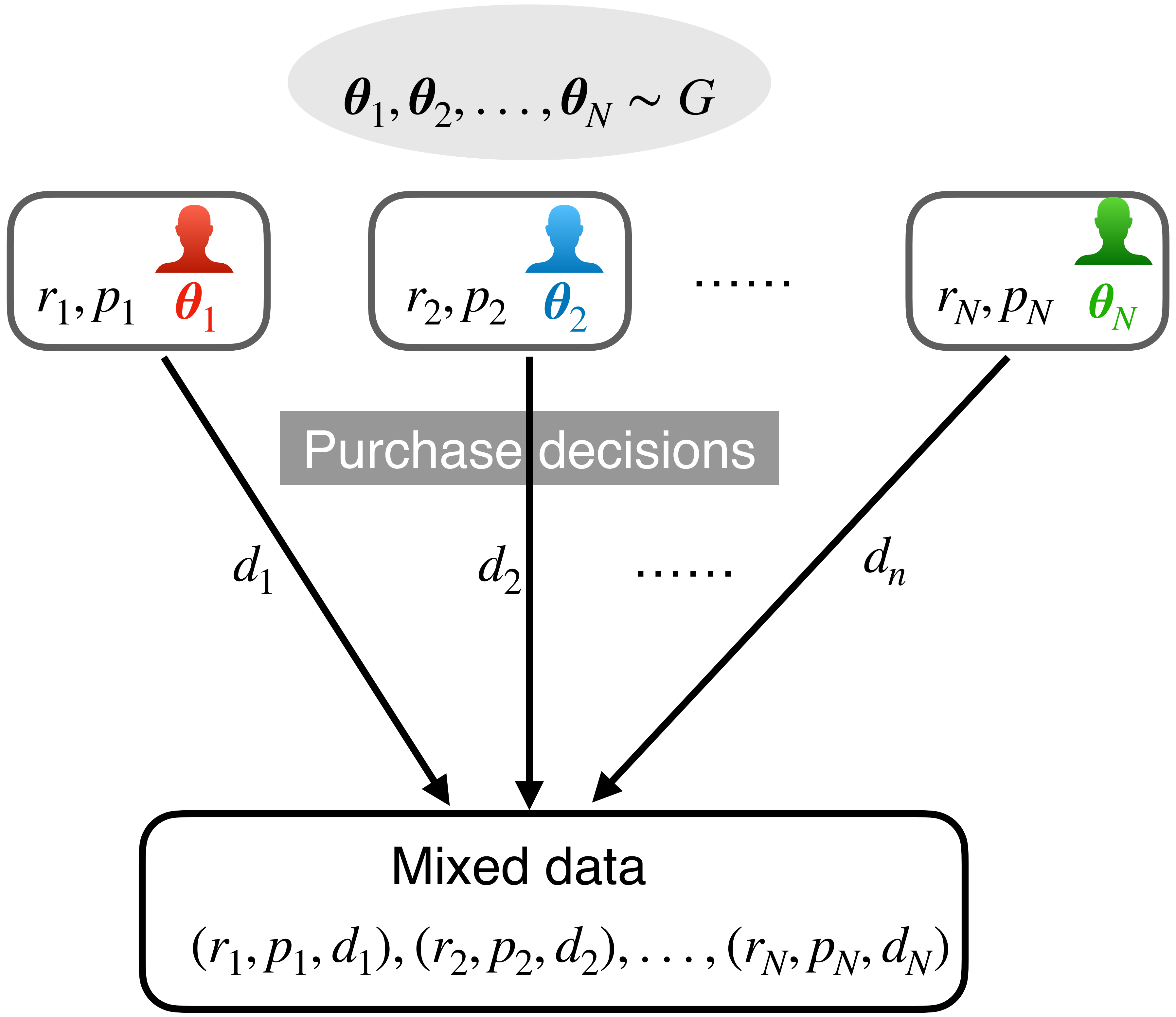

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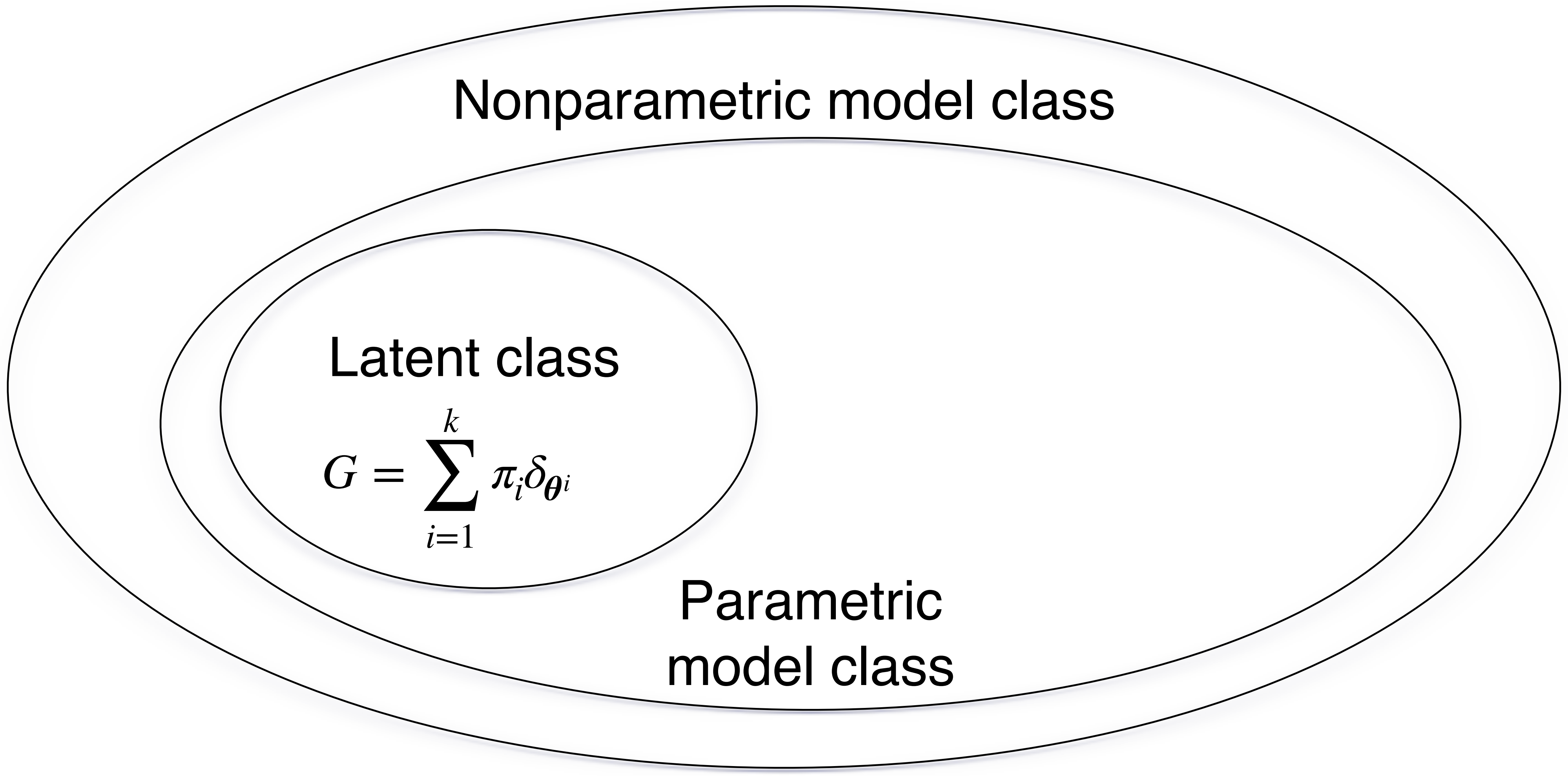

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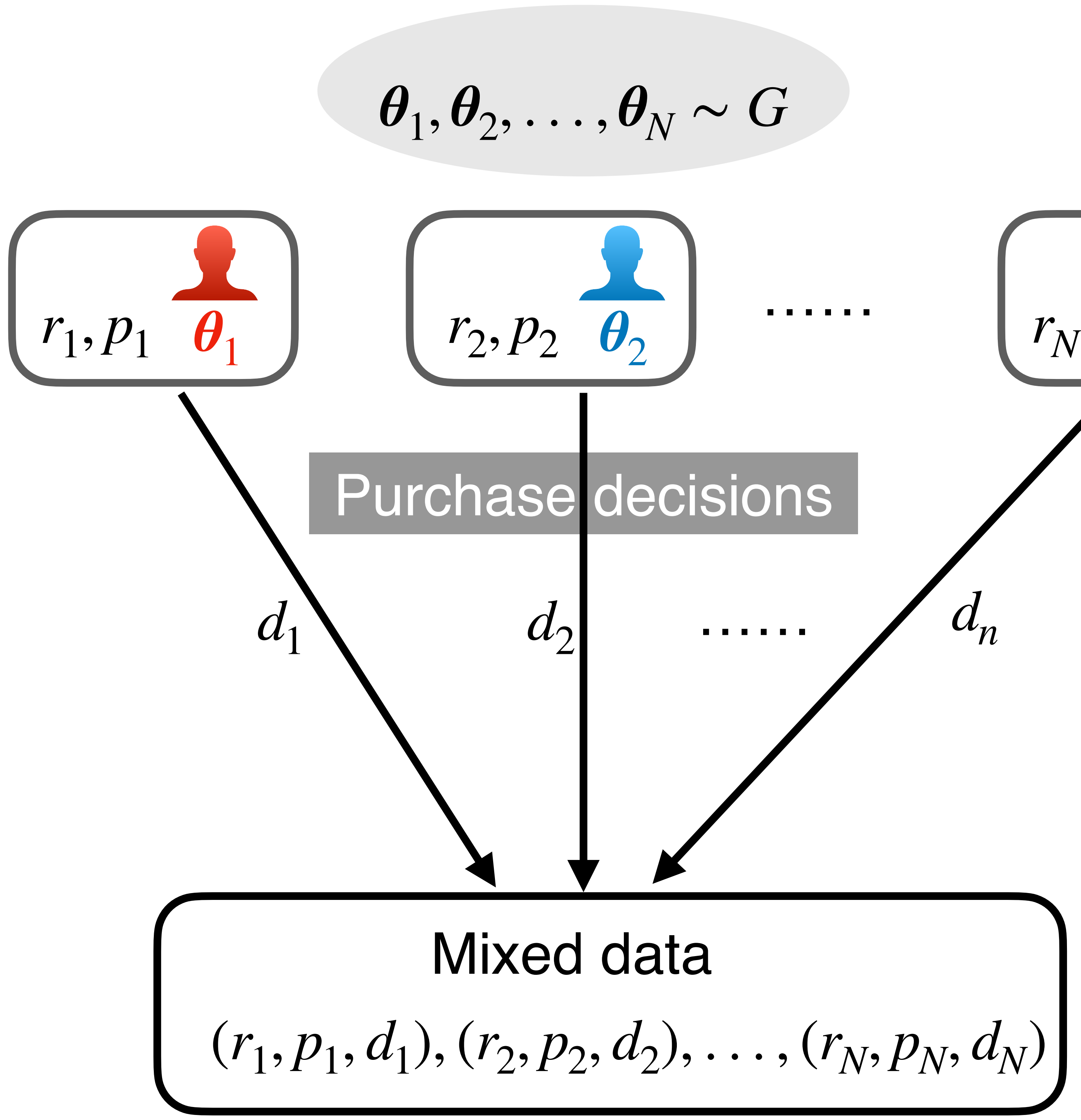

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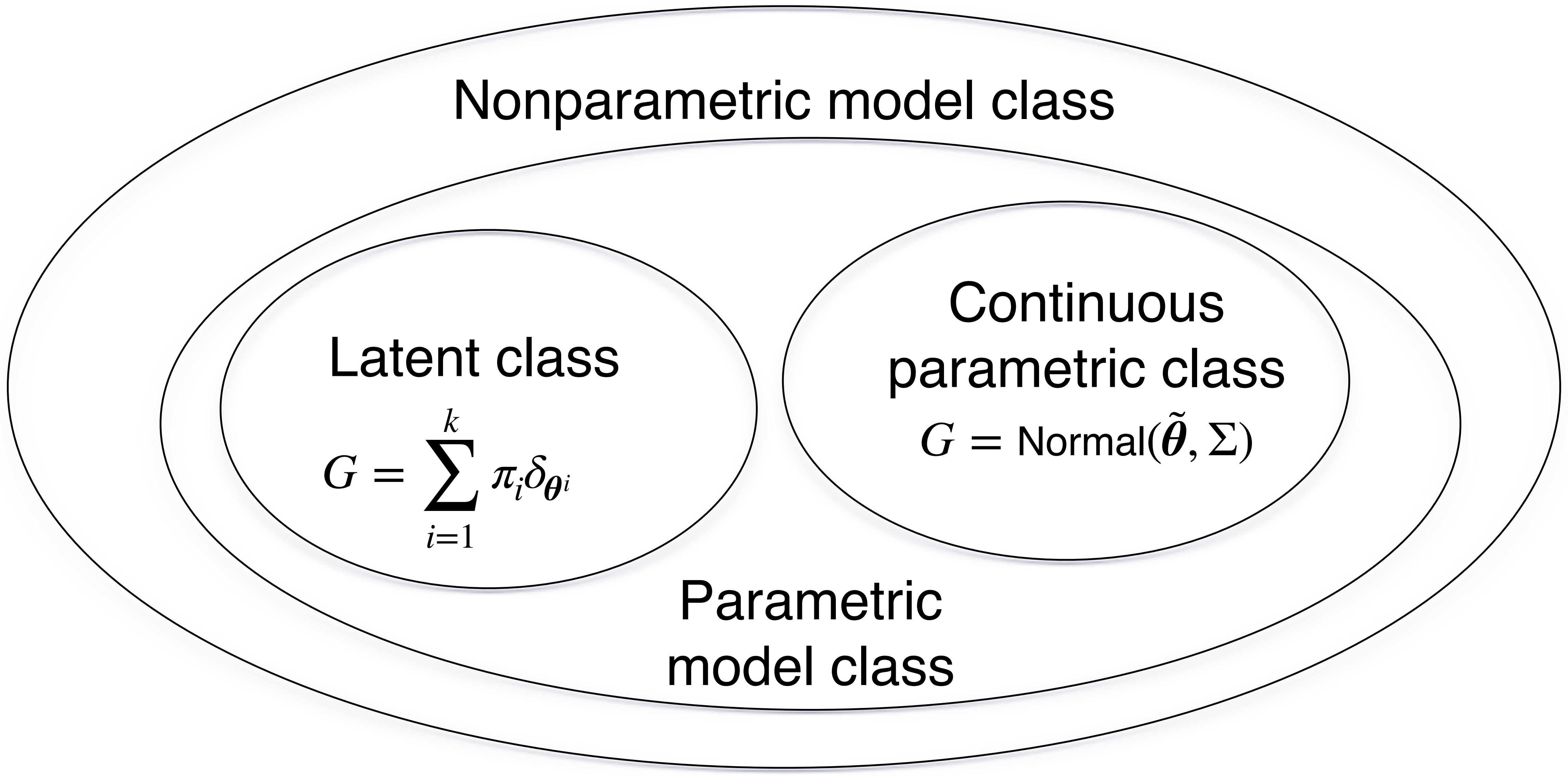

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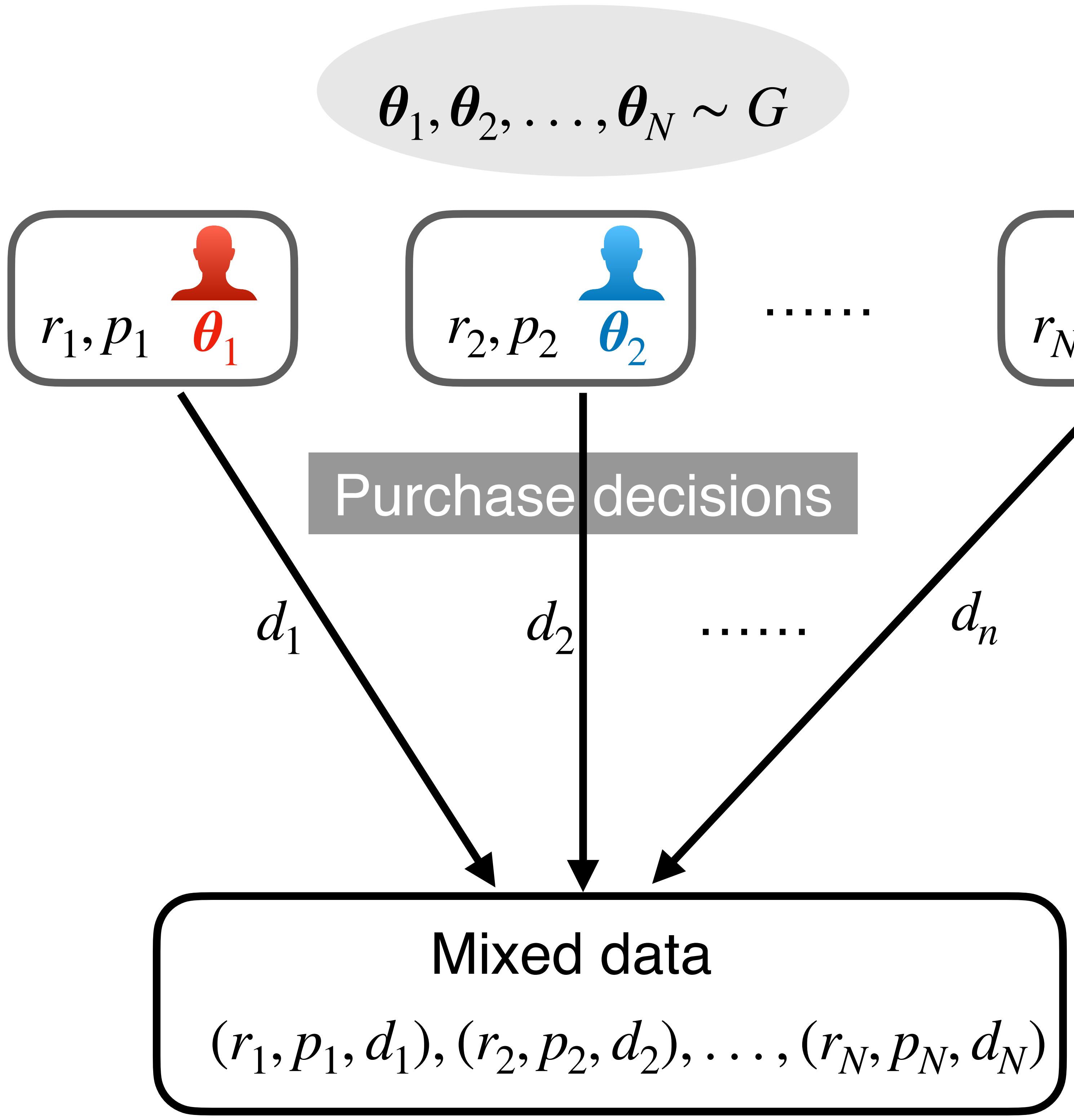

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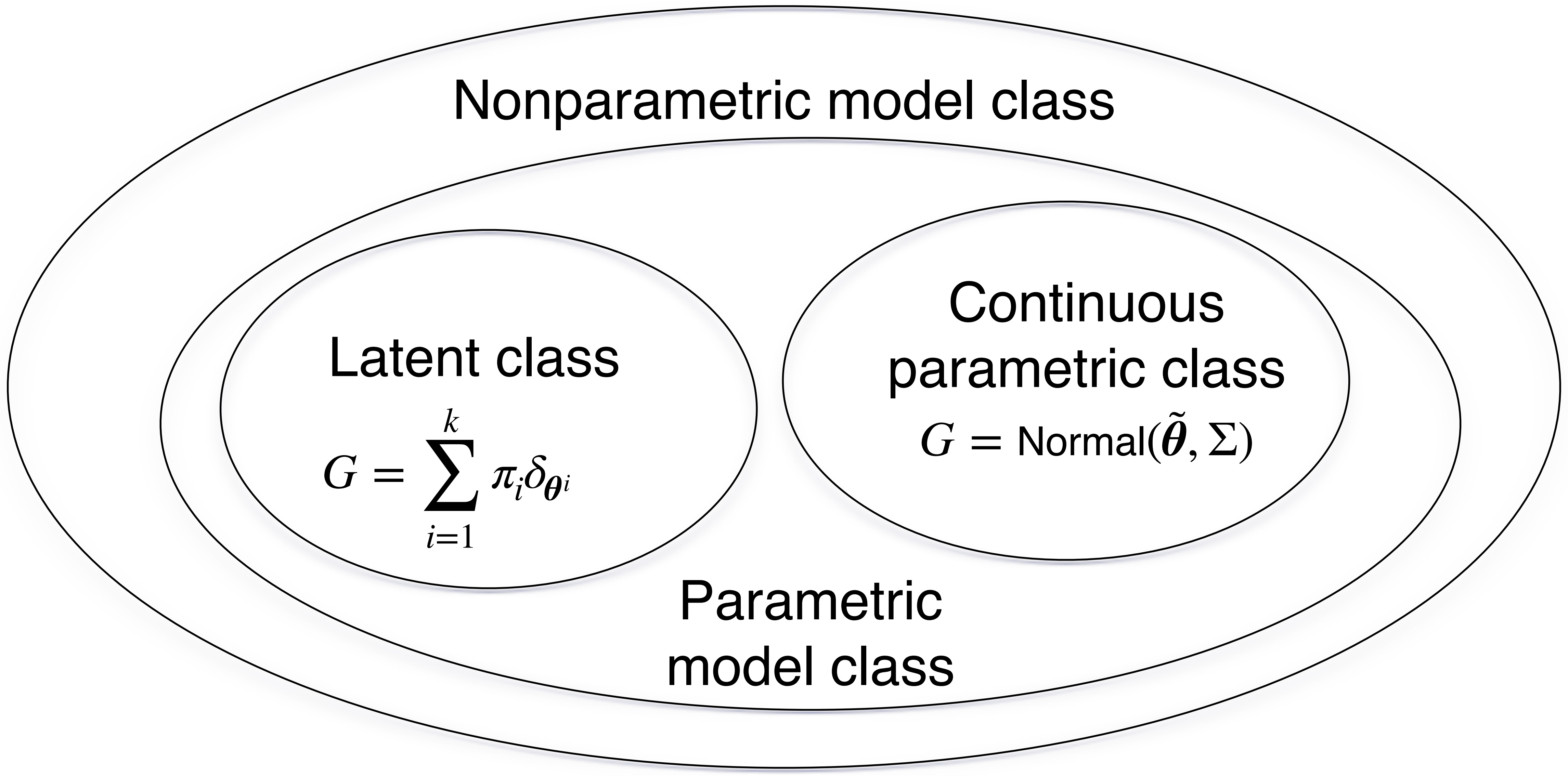

Our proposal

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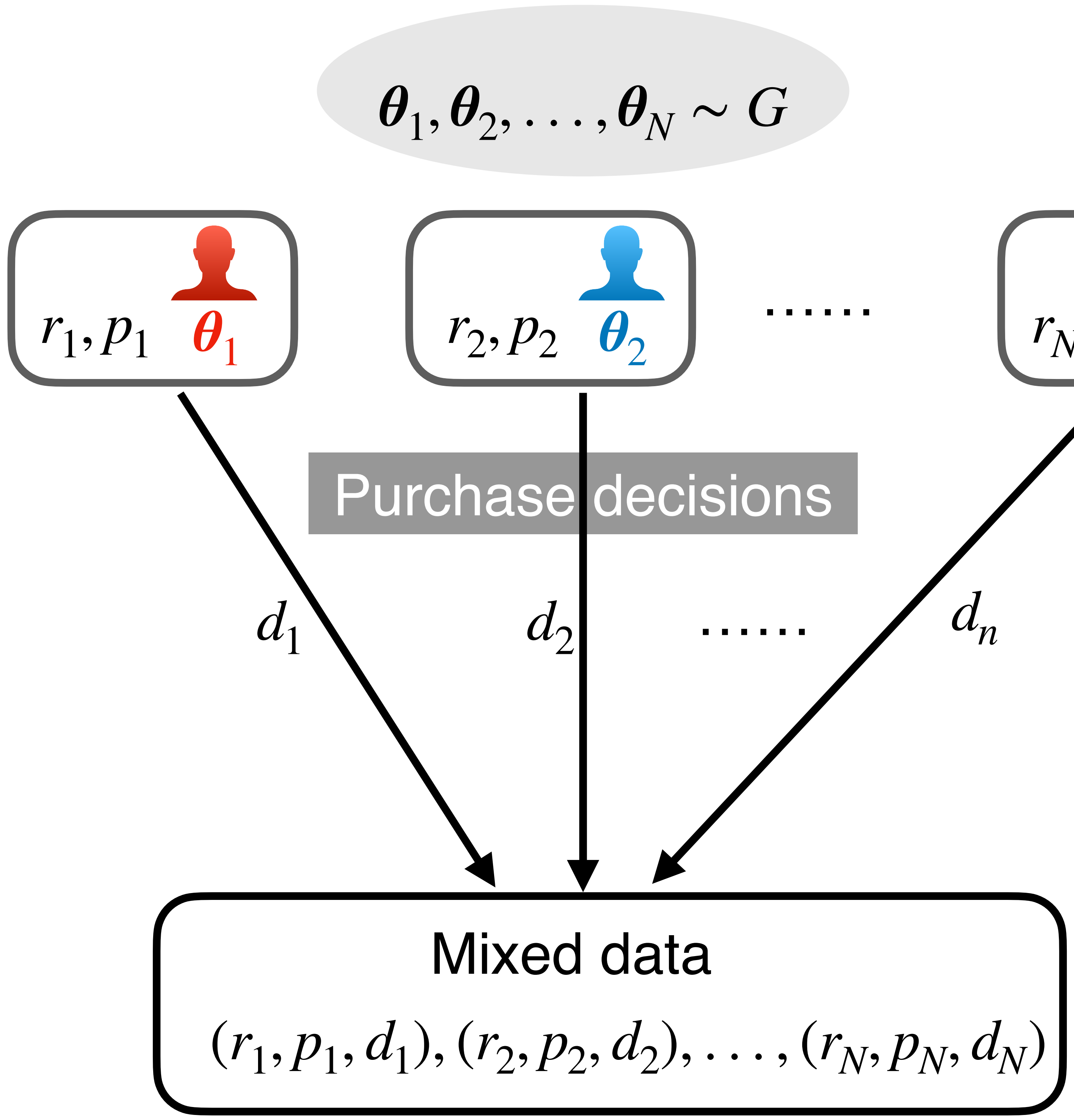

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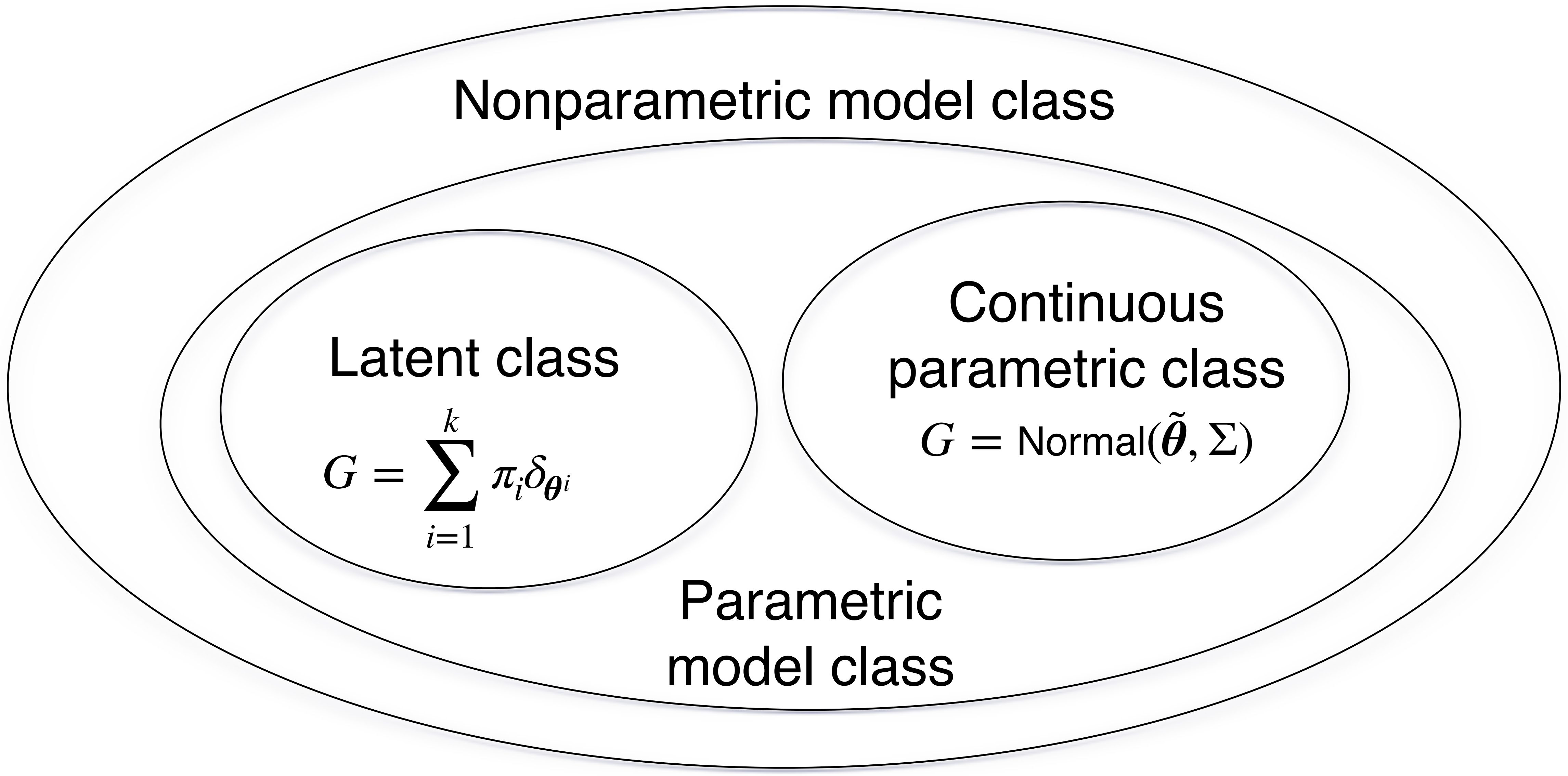

Our proposal

- Nonparametric maximum likelihood estimator (NPMLE)

NPMLE of Consumer Distribution G

"A Nonparametric Maximum Likelihood Approach to Mixture of Regression." R&R at *Journal of the American Statistical Association.* **H. Jiang**, *A. Guntuboyina.*

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Theorem (informal)
NPMLEs exist and there exists an NPMLE that is supported on at most N components.

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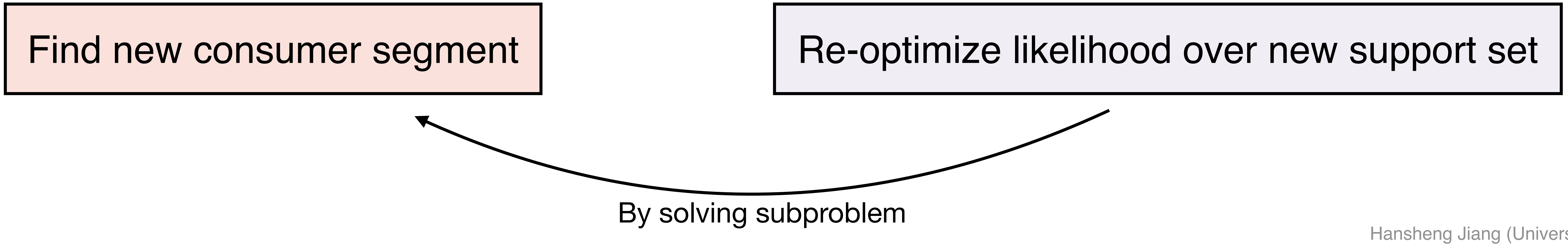

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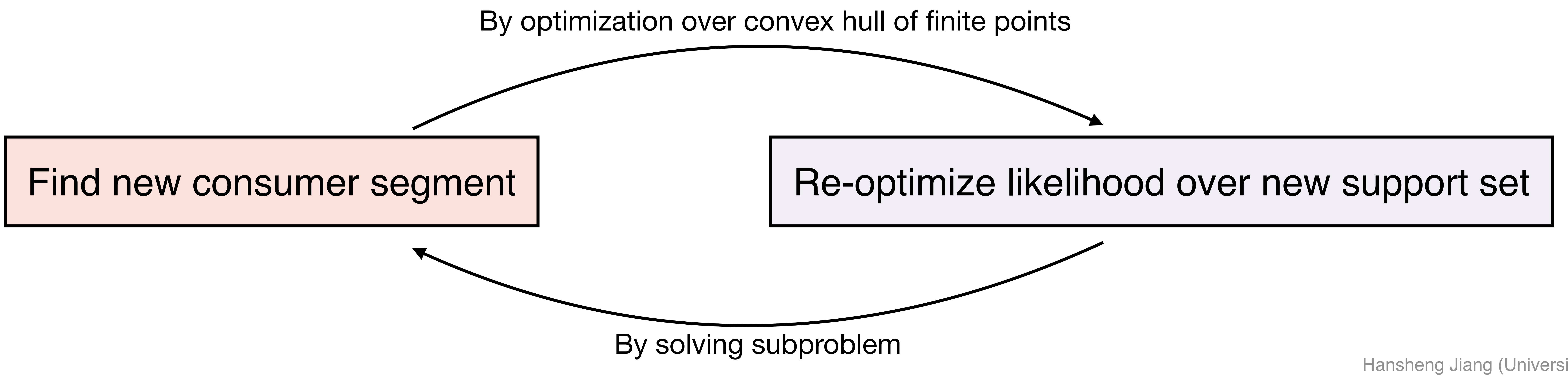

Contributions

Formulate the heterogeneous consumer reference effects model in the individual level

Propose a nonparametric statistical method for extracting consumer heterogeneity from transaction data

Provide computational algorithm for optimal pricing policies and establish the sub-optimality of constant policies

Apply to real-world data from retailing platform JD.com and show that the proposed approach leads to significant improvement in revenue

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  - Find the price sequence $\{p_t\}_{t=1}^{\infty}$ that maximizes the long-term discounted revenue

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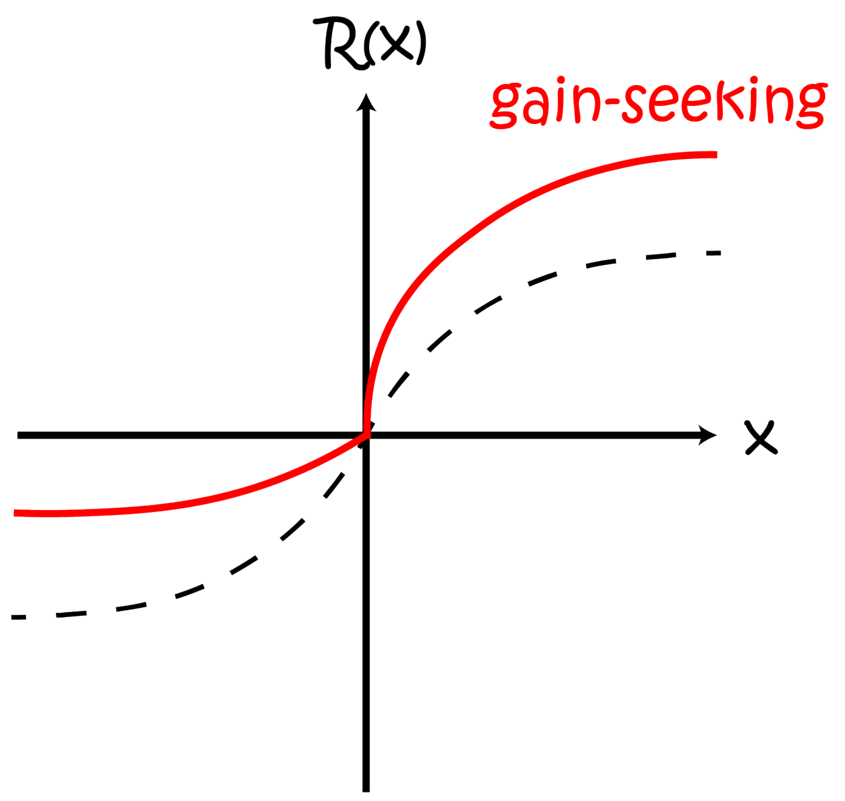

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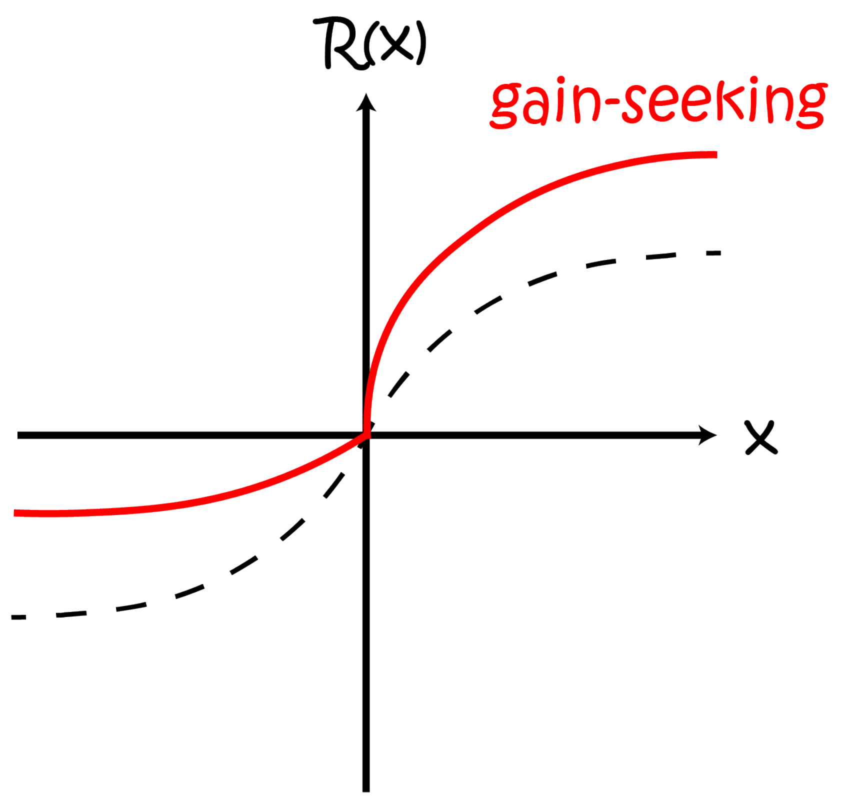

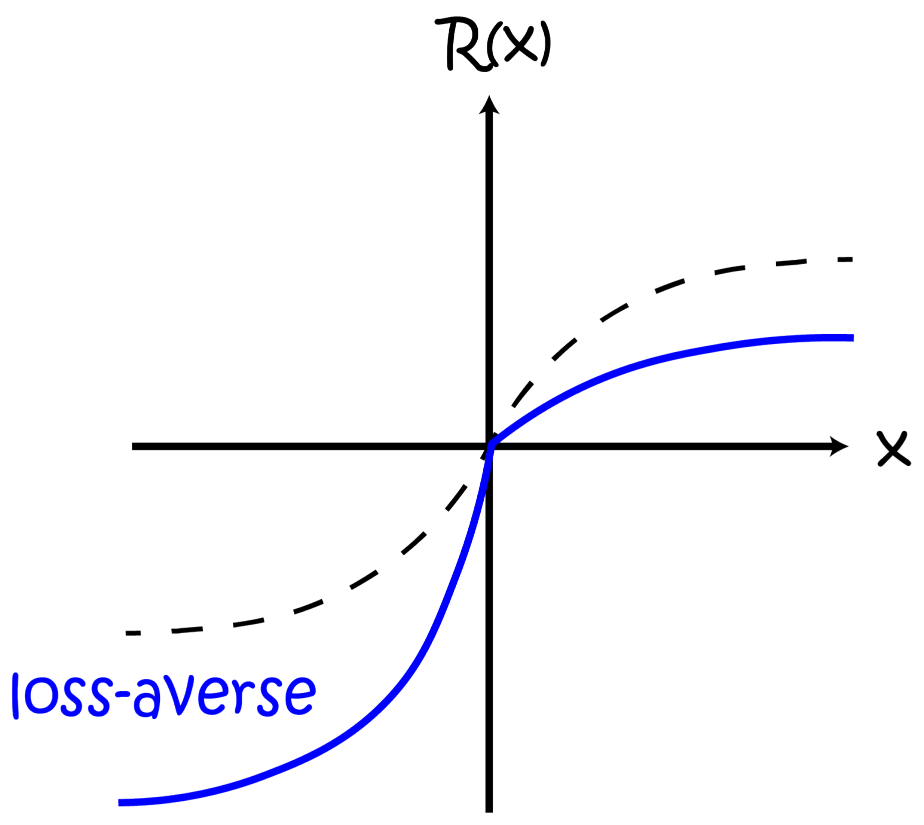

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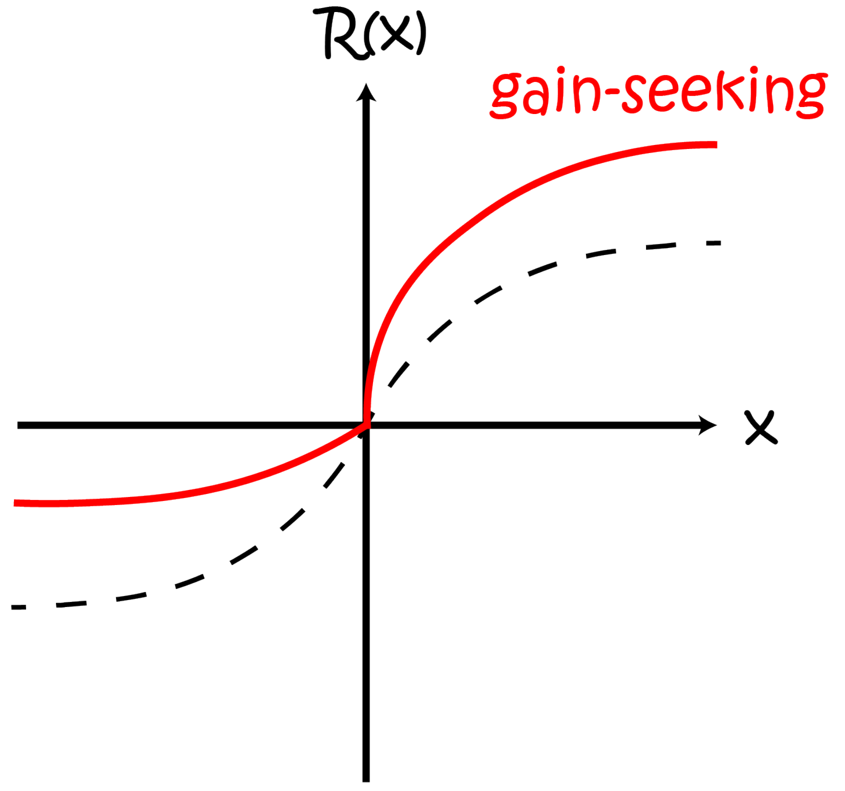

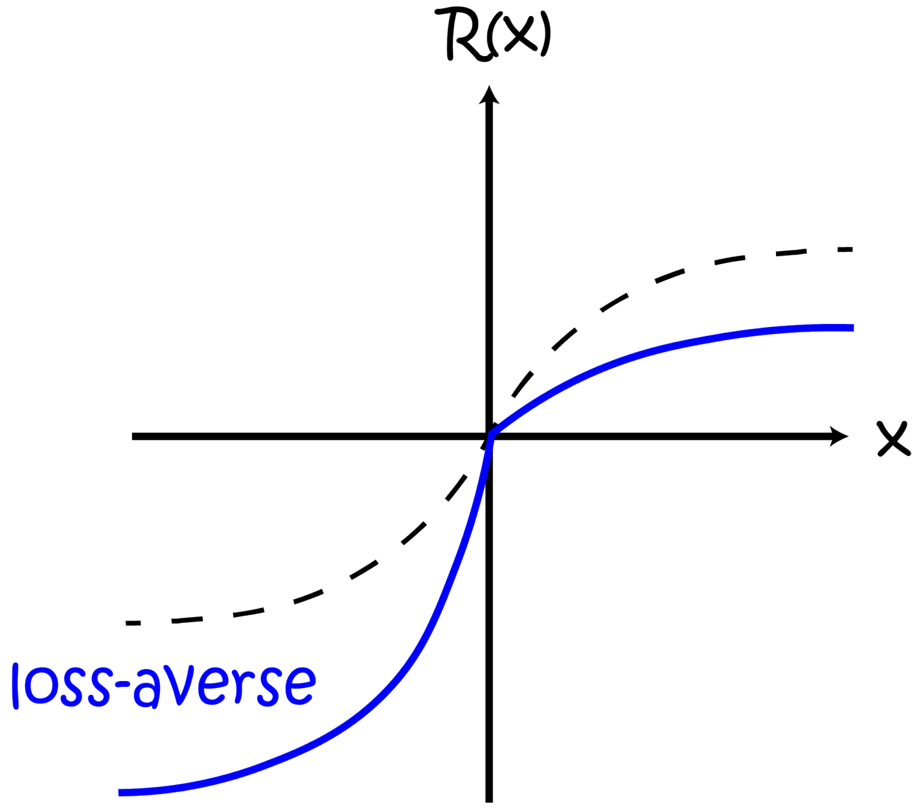

Question: do similar pricing structures hold in our individual consumer model?

Sub-optimality Results

Theorem (Sub-optimality of constant pricing policy, informal)
For sufficiently large c_{-}, the constant pricing policy is **not** optimal even if $c_{+} \leq c_{-}$ (loss-averse or neutral) .

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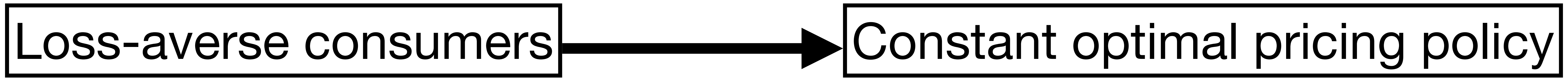

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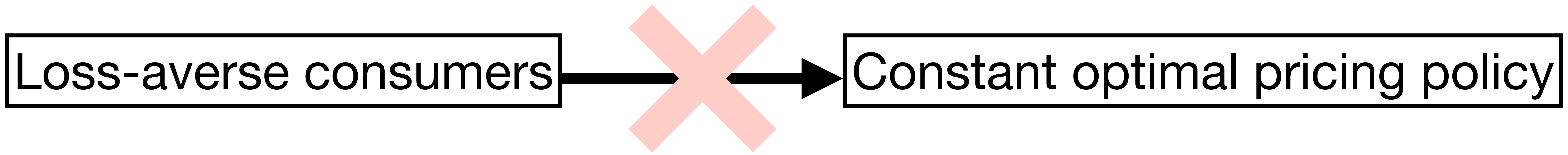

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- View as dynamic programming

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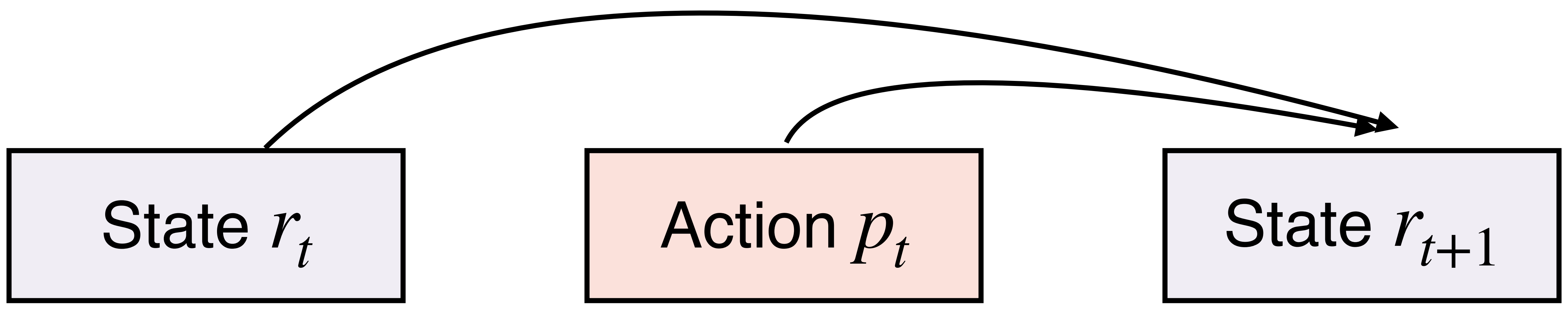

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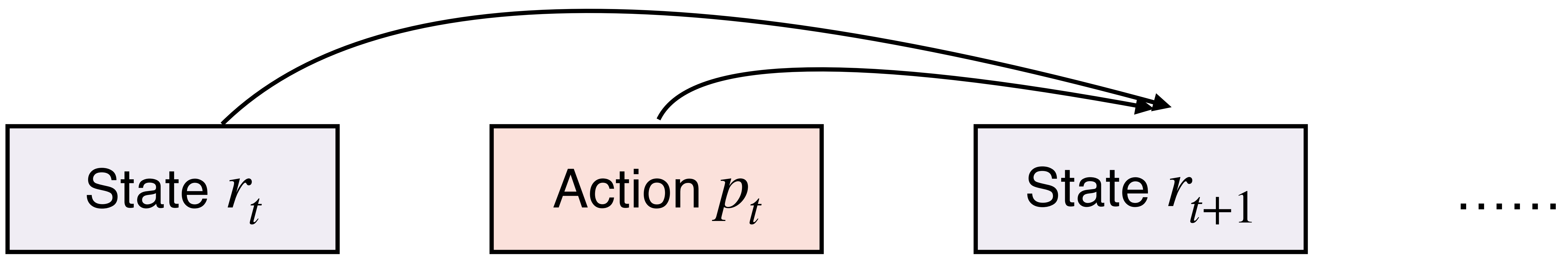

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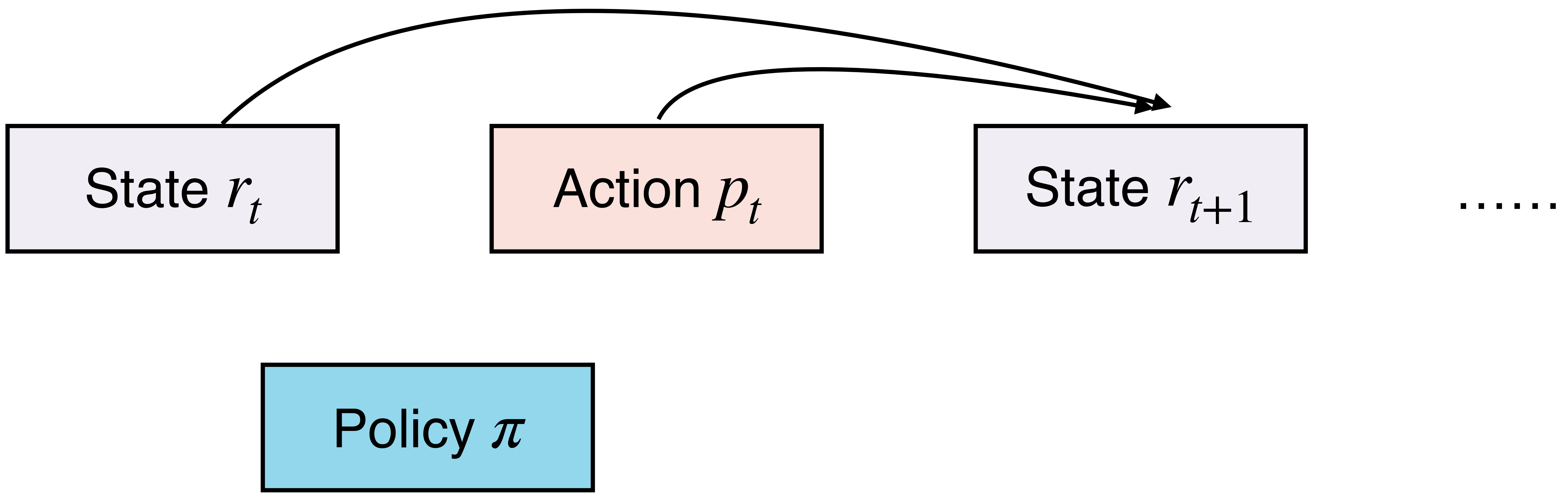

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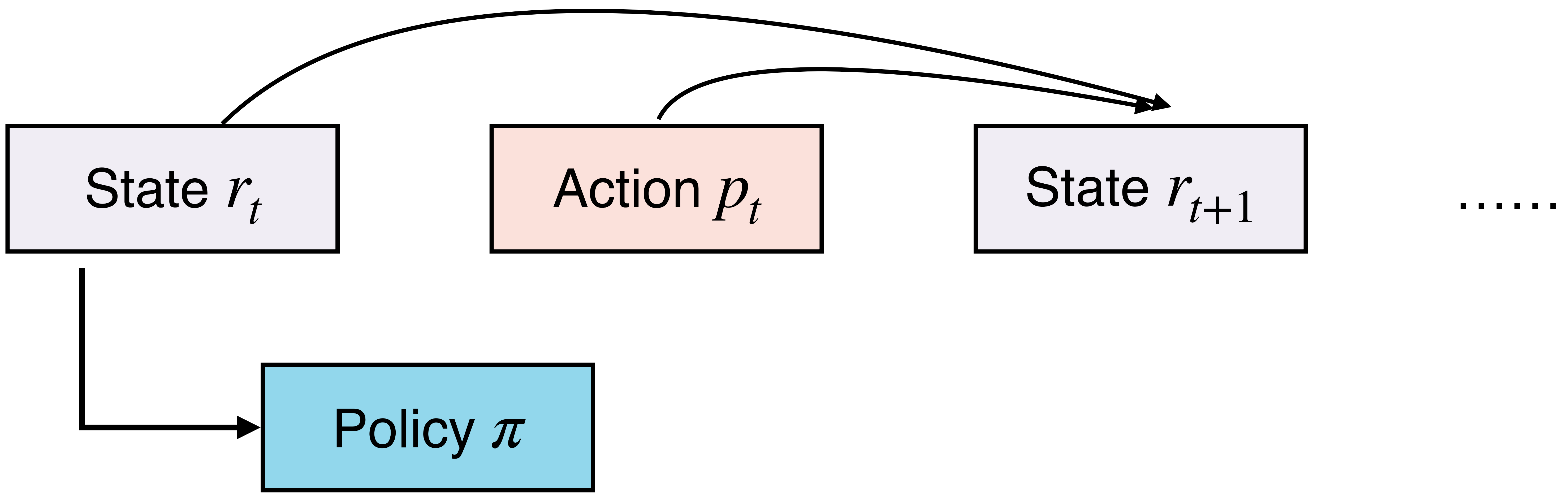

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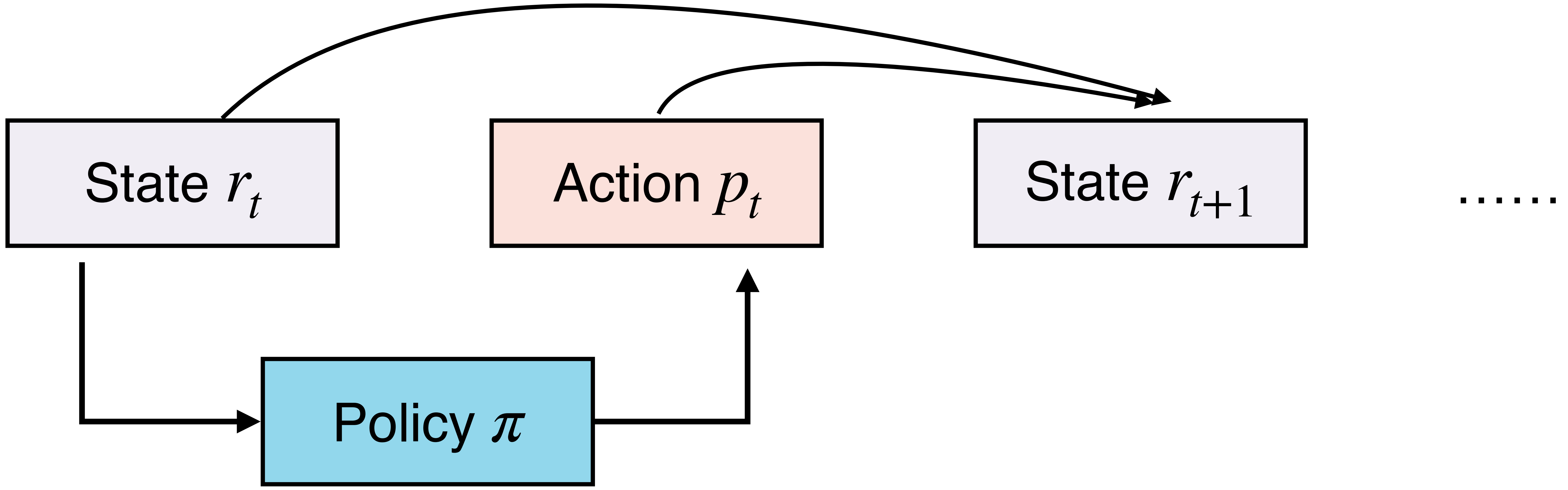

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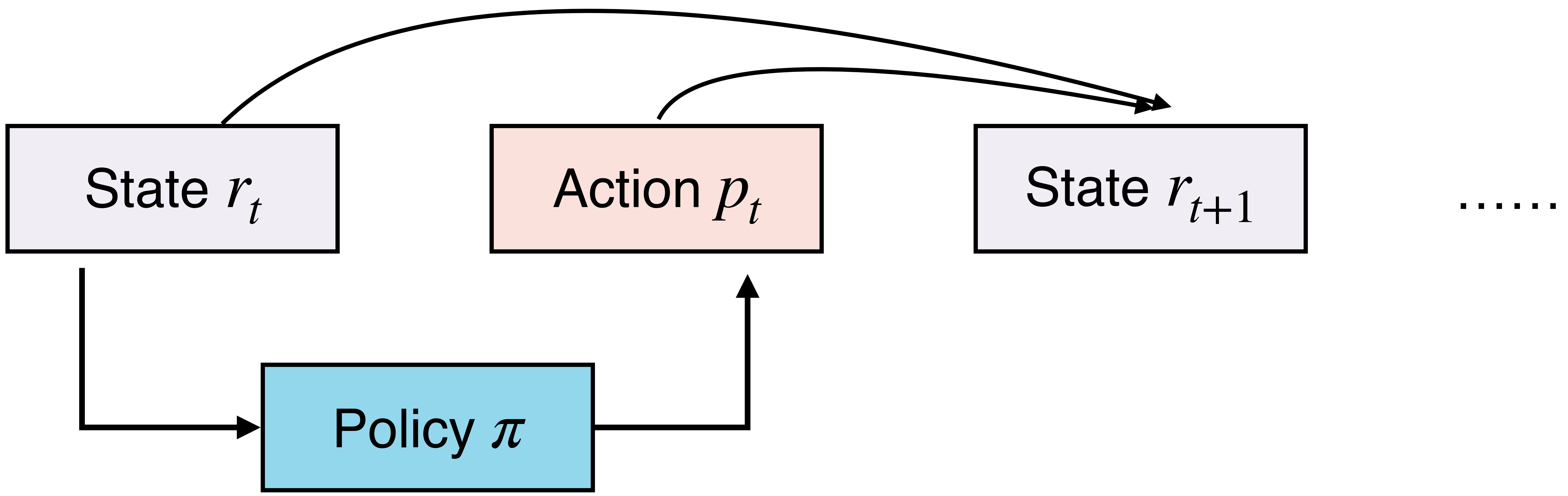
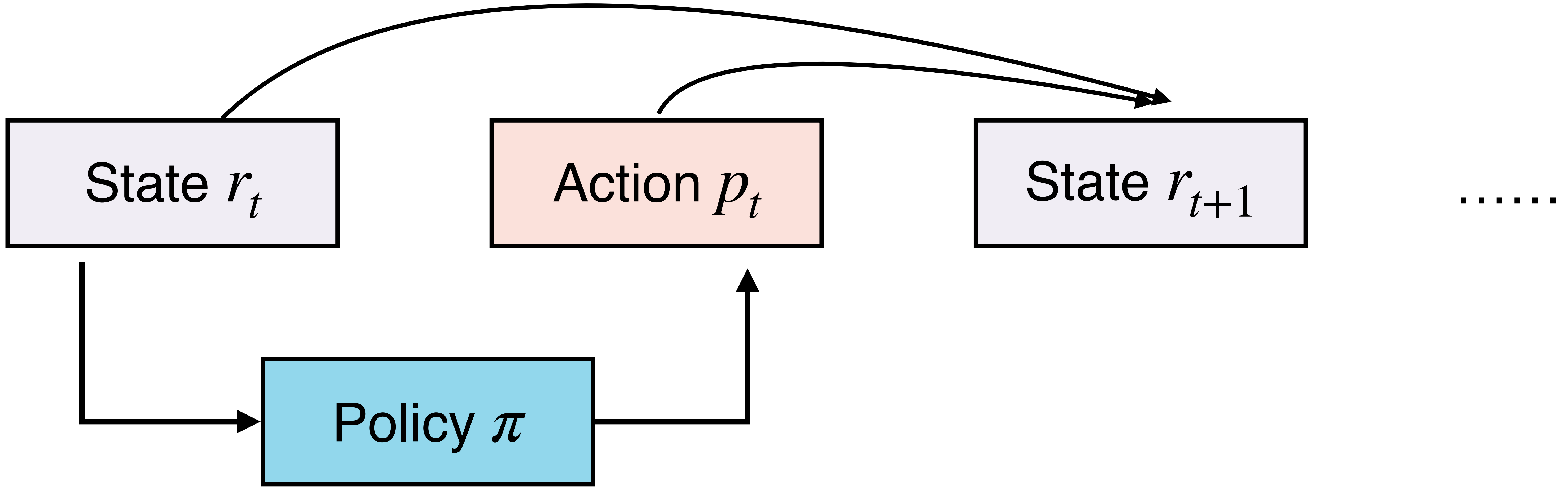


- Value function: long-term discounted revenue

Optimizing Long-Term Revenue

- View as dynamic programming



- Value function: long-term discounted revenue

Theorem (Discretization guarantee, informal) The difference of the optimal long-term discounted revenue and its counterpart under discretization is bounded by

$$0 \leq V^\star(r) - V^\star_\epsilon(r) \leq O(\epsilon) \, .$$

Computing Optimal Pricing Policy

Computing Optimal Pricing Policy

Pricing Optimization

Computing Optimal Pricing Policy

Pricing Optimization

$$V(r_0) = \max_{\{p_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t \, p_t \mathrm{P}^G(r_t, p_t)$$

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$$\text{s.t. } r_t = (1-\alpha)p_{t-1} + \alpha r_{t-1}$$

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Modified policy iteration algorithm

Initialize $V^0 = 0, k = 1$

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Repeat

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Generate new pricing policy π_k based on value function V^{k-1}

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Approximate policy evaluation

Calculate the value function V^k according to policy π_k

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Modified policy iteration algorithm

Initialize $V^0 = 0, k = 1$

Repeat

- **Policy improvement**
  - Generate new pricing policy π_k based on value function V^{k-1}
- **Approximate policy evaluation**
  - Calculate the value function V^k according to policy π_k
- $k \leftarrow k + 1$

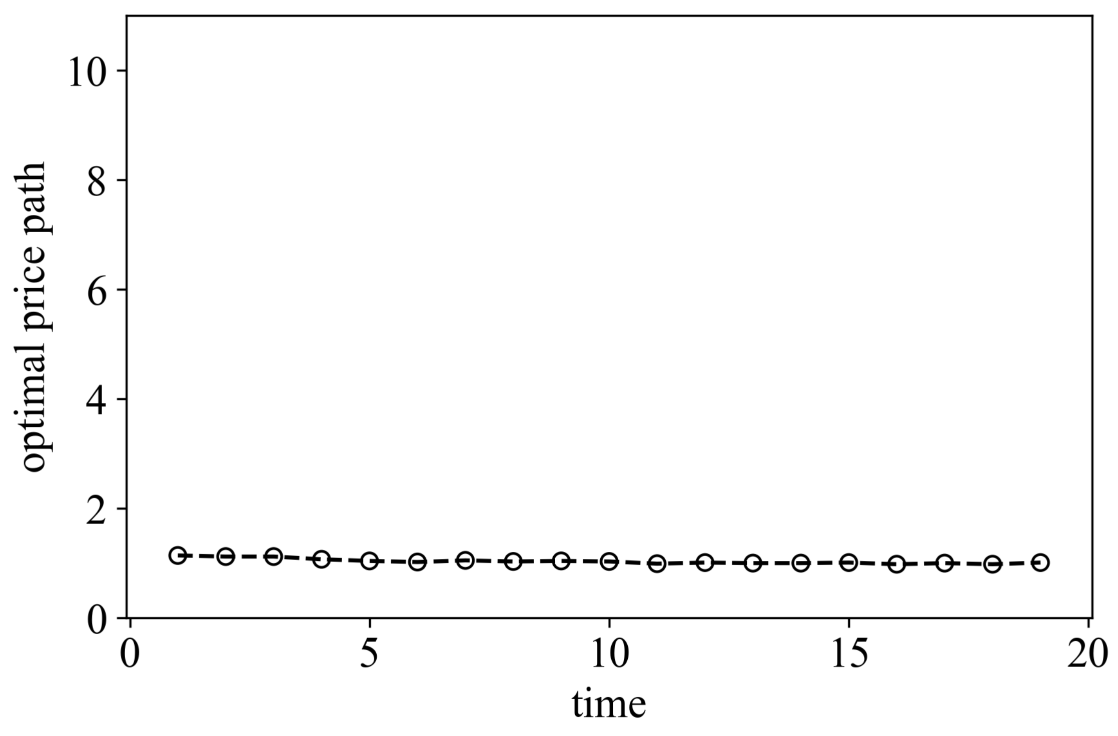
Until convergence

Example: Two Market Segments

Example: Two Market Segments

(a) Homogeneous, consumer A only

$(a_A, b_A, c_{A+}, c_{A-}) = (2,2,0.2,0.2)$



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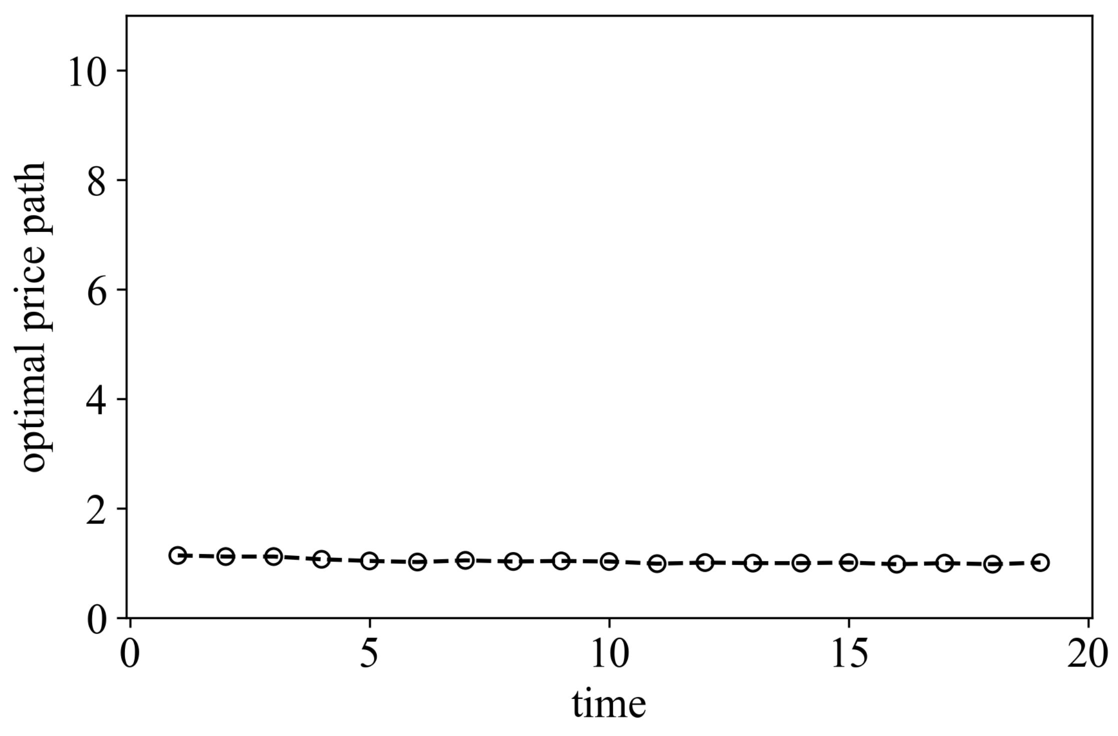

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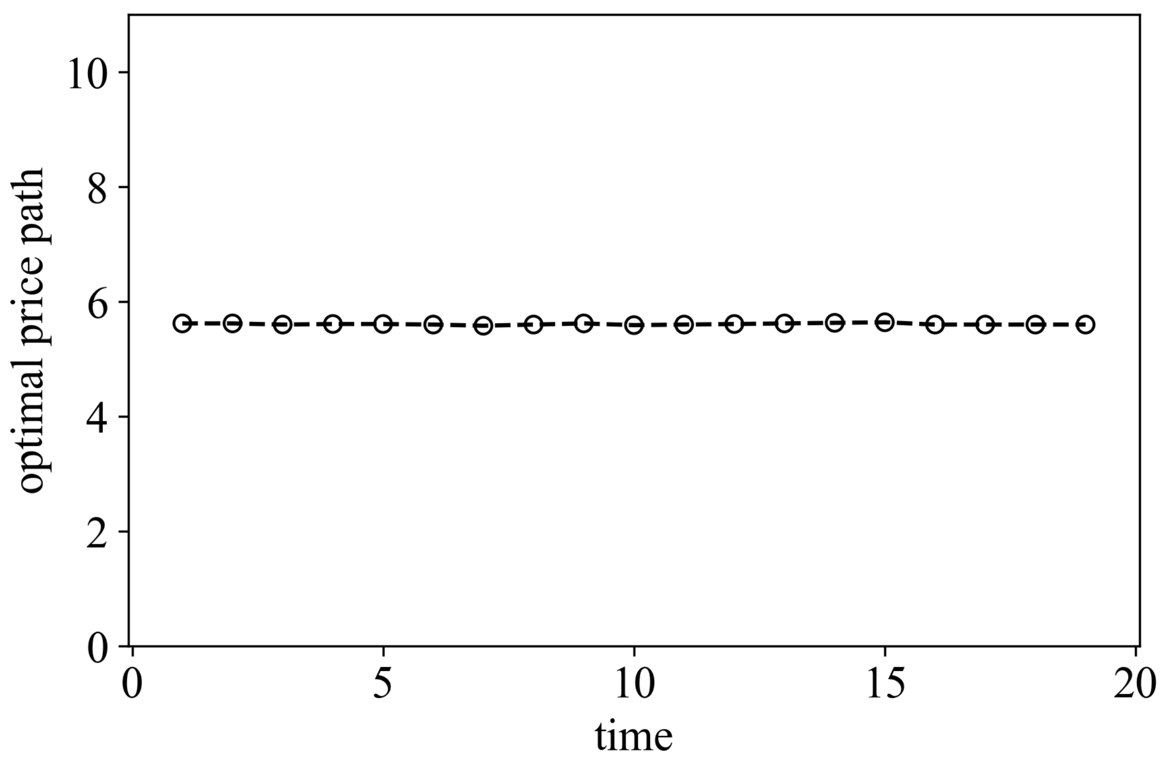

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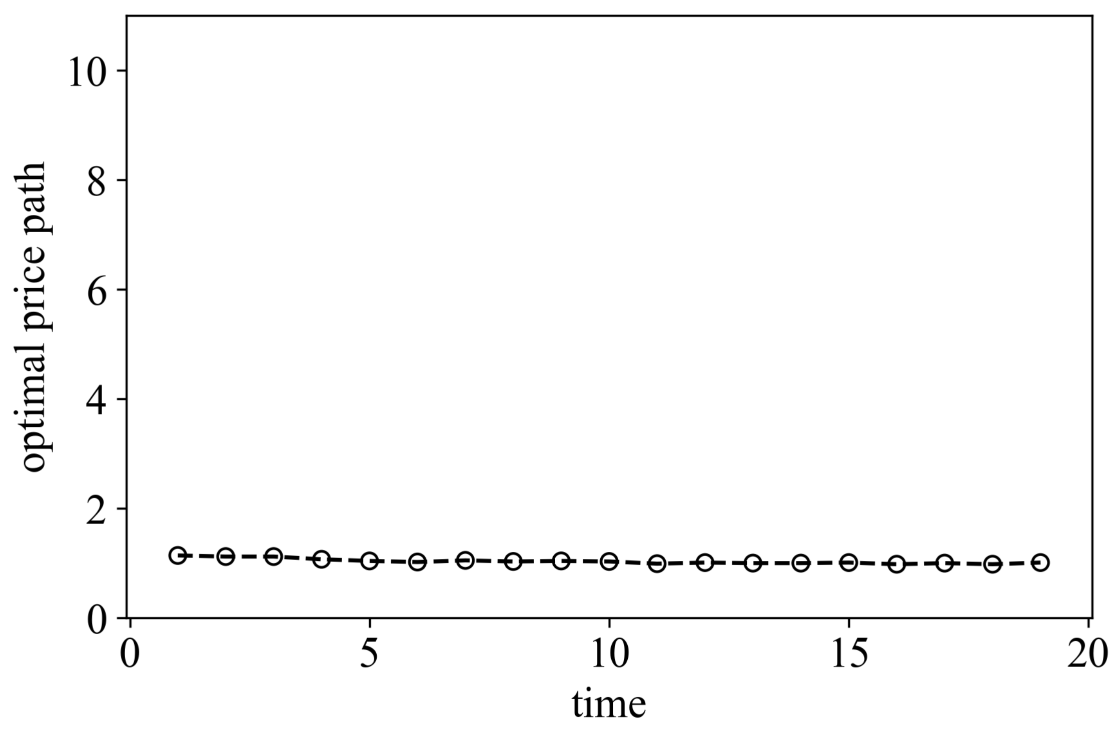

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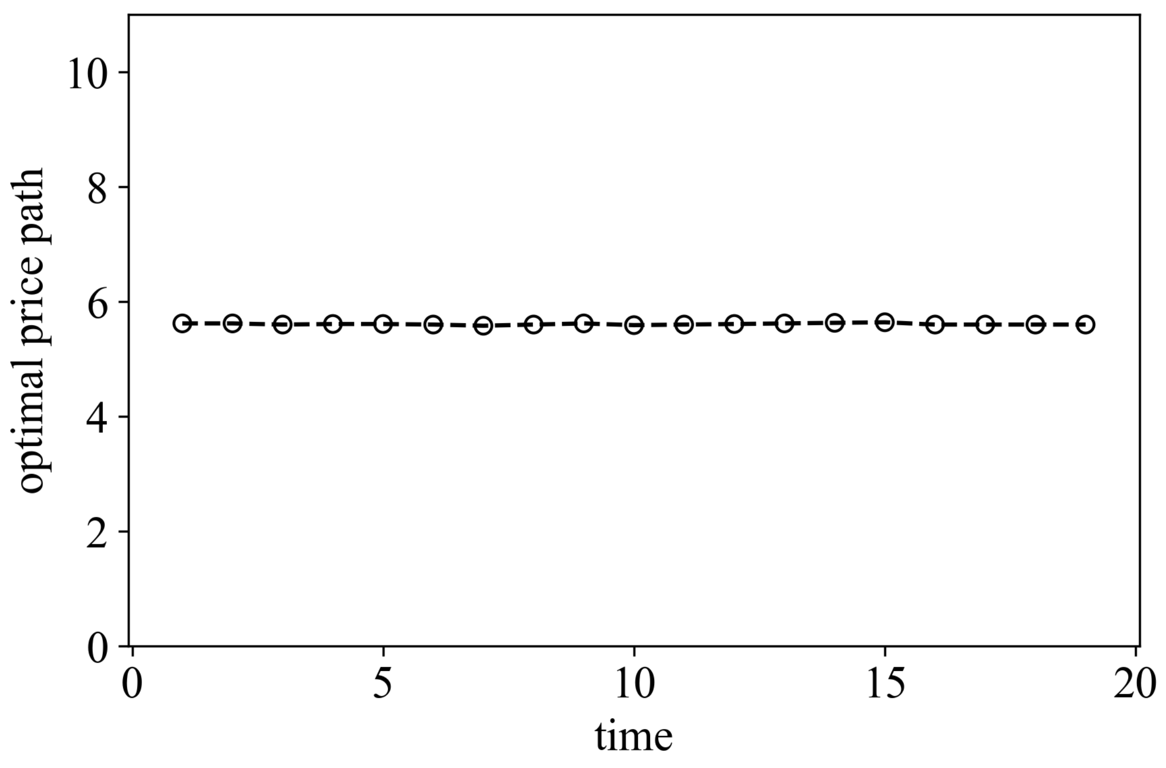
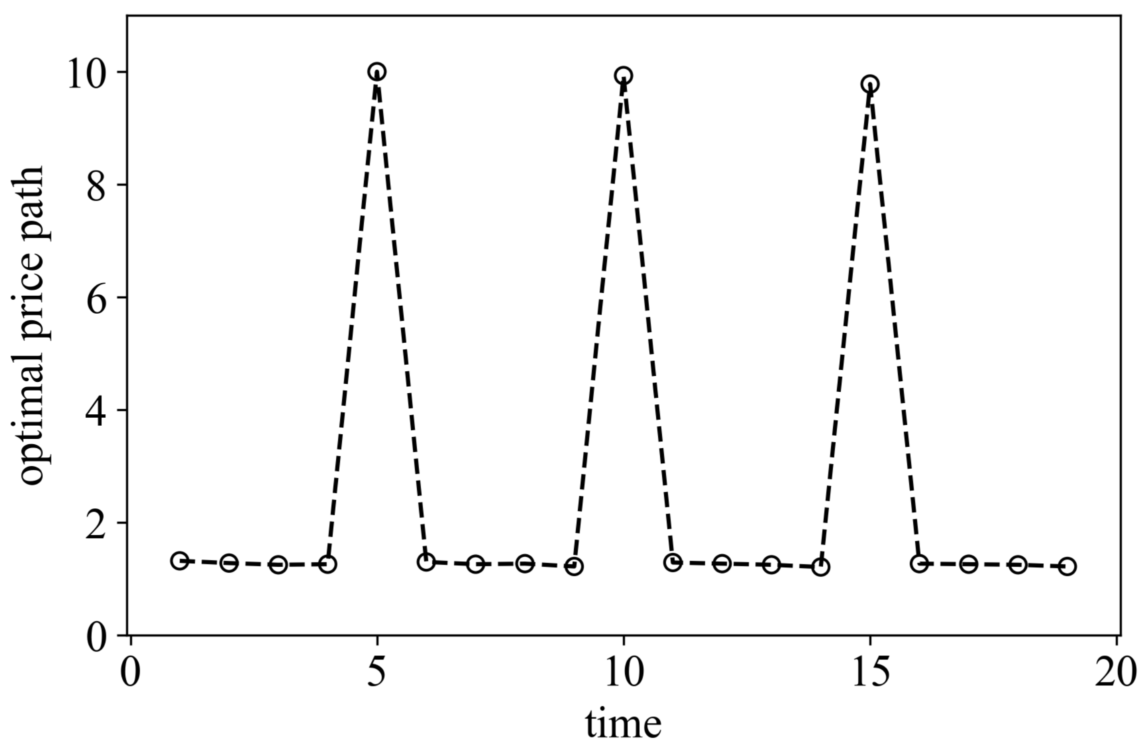


(c) Heterogeneous, 50% consumer A, 50% consumer B



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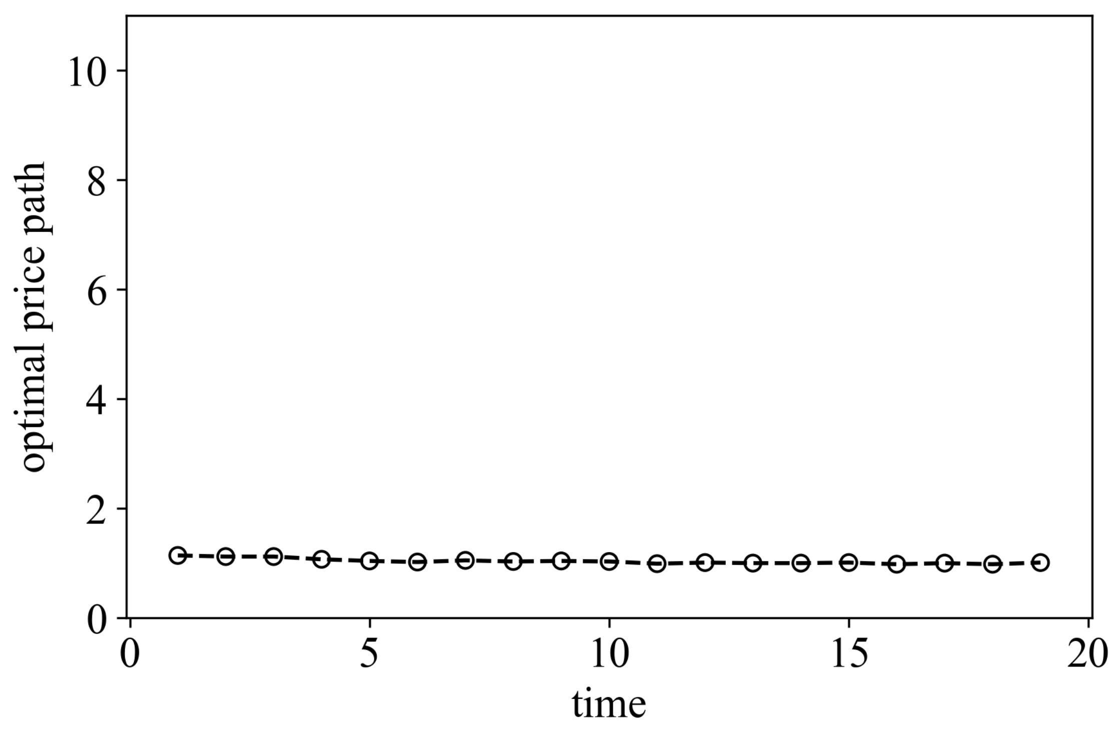

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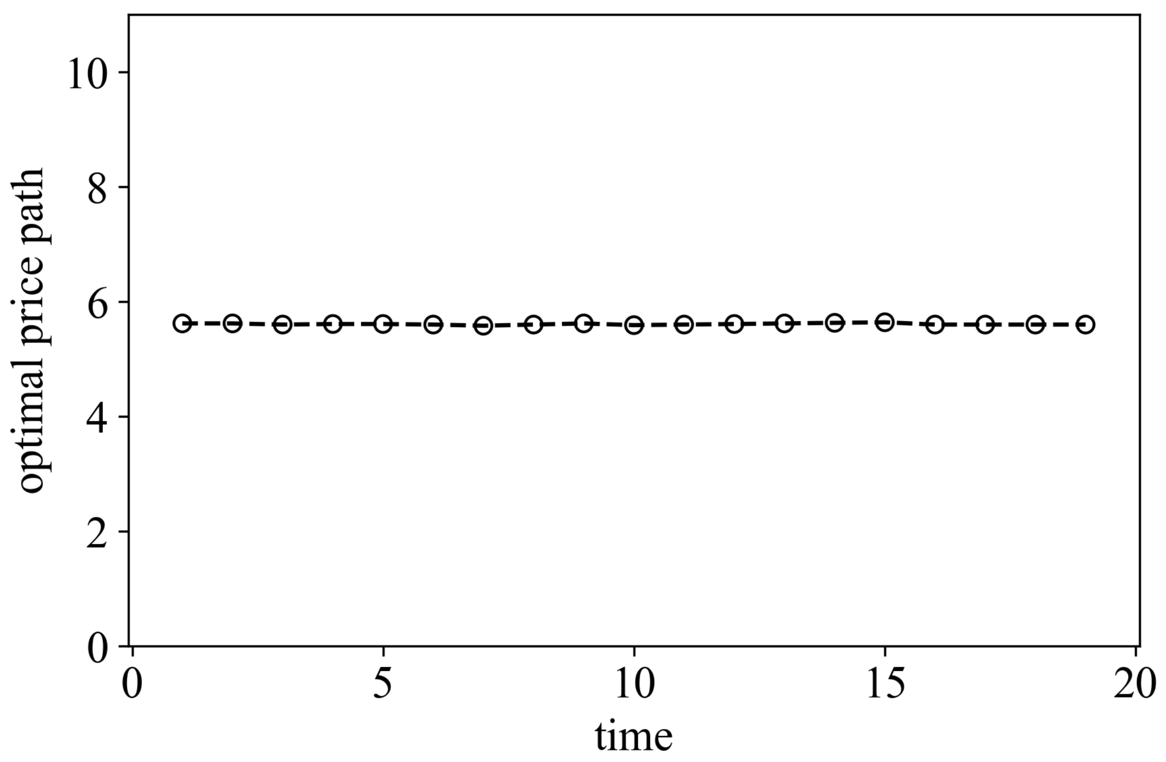

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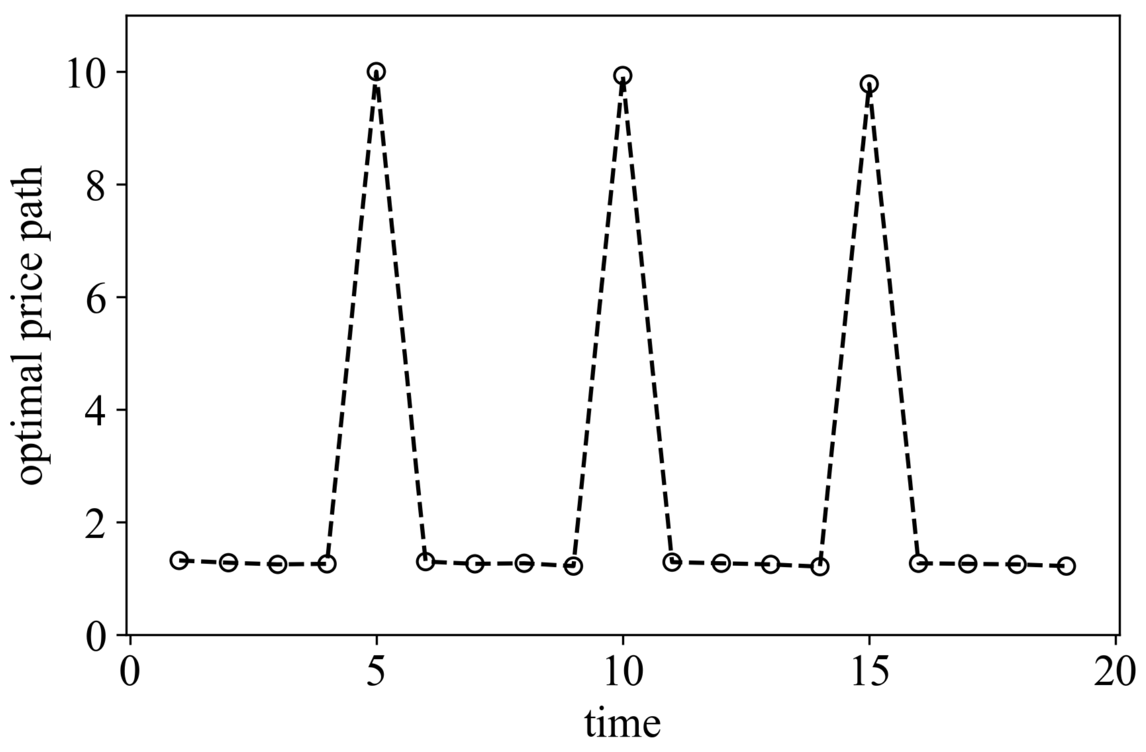
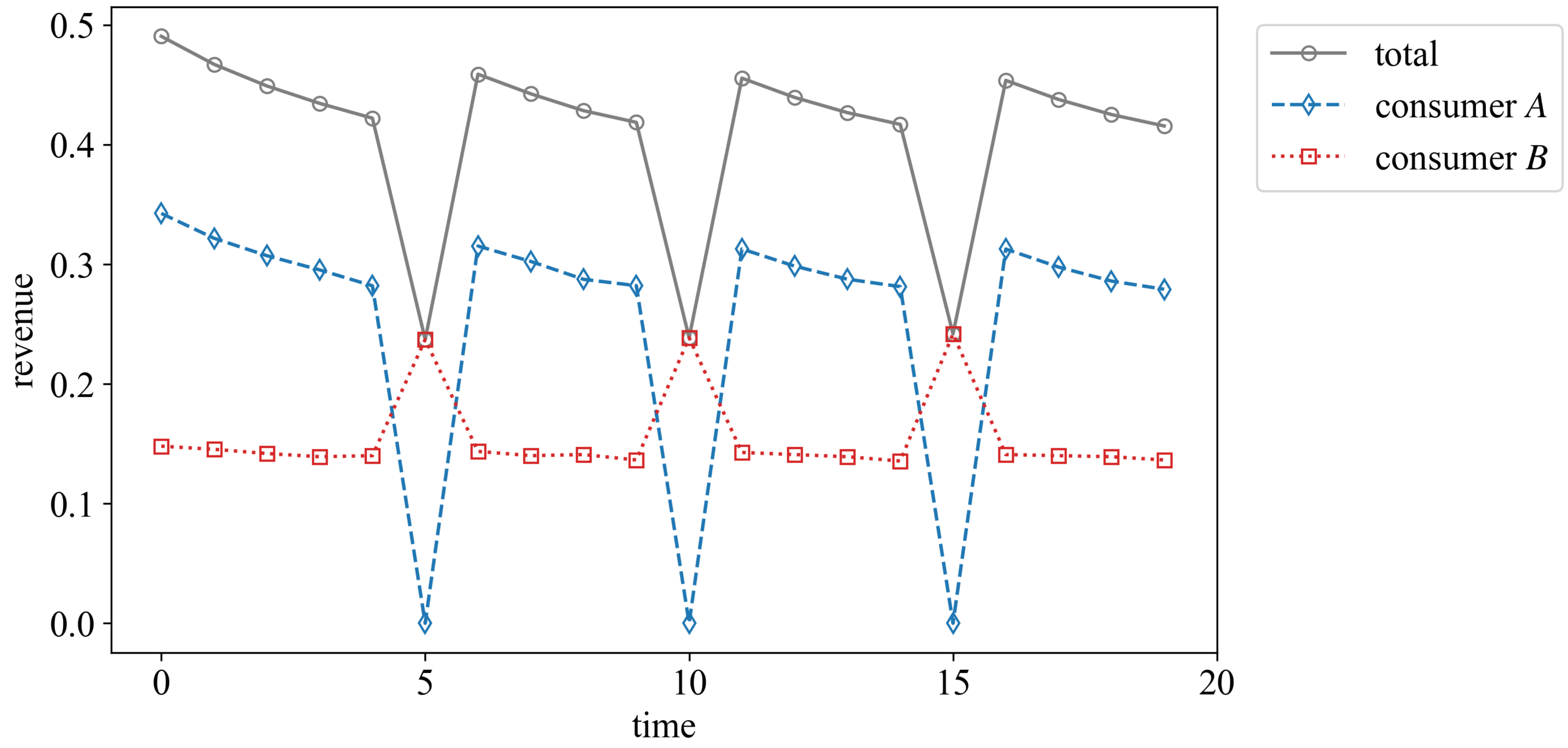


(d) Per period revenue



Example: Two Market Segments

Constant optimal pricing + constant optimal pricing ≠ constant optimal pricing

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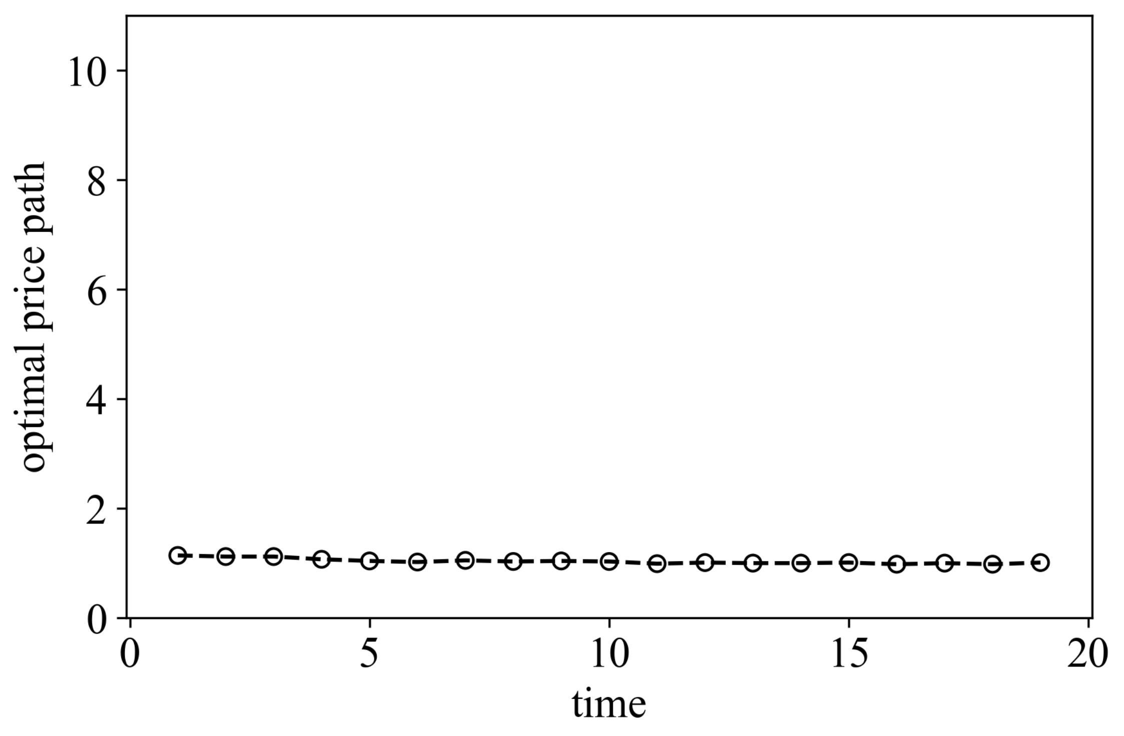

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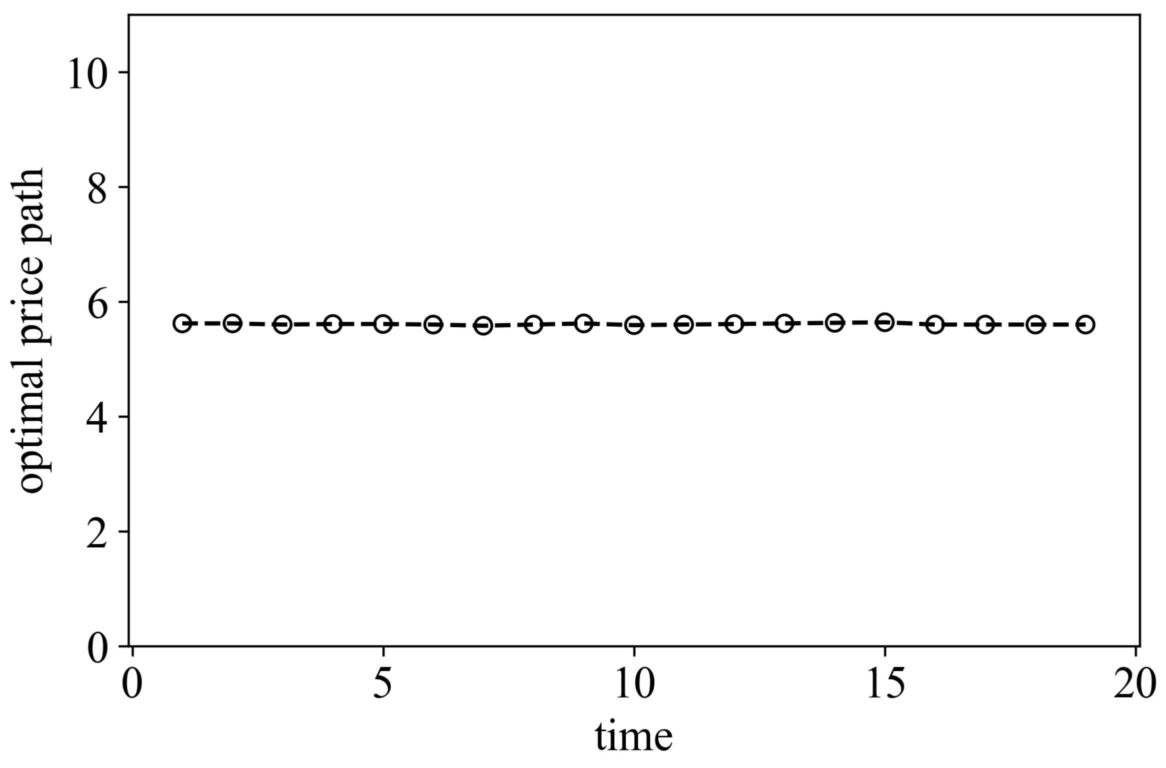

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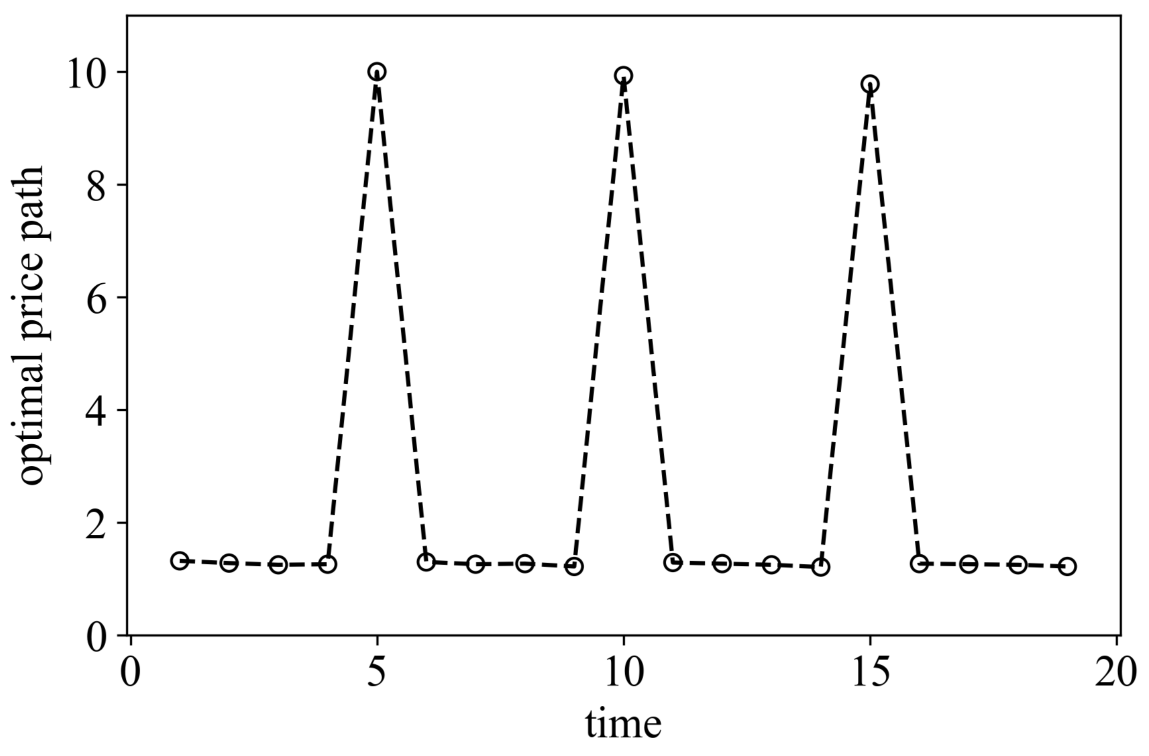

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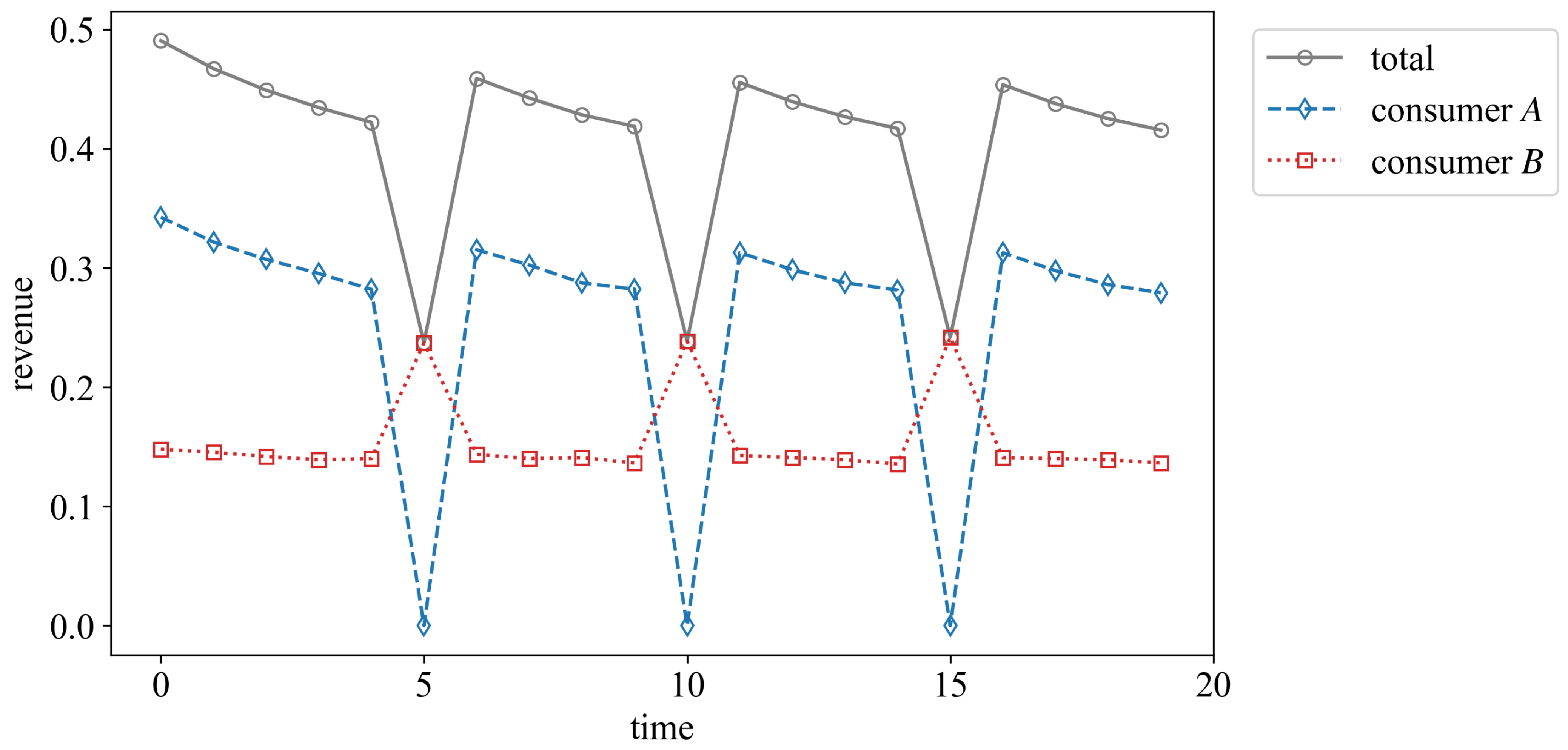

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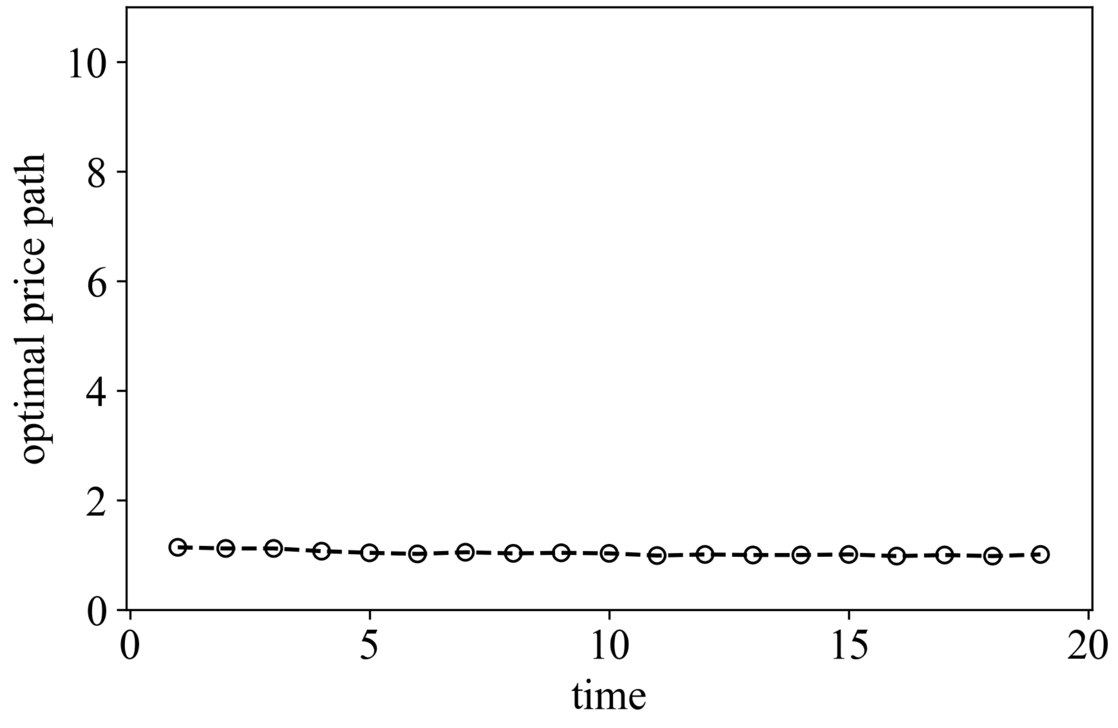

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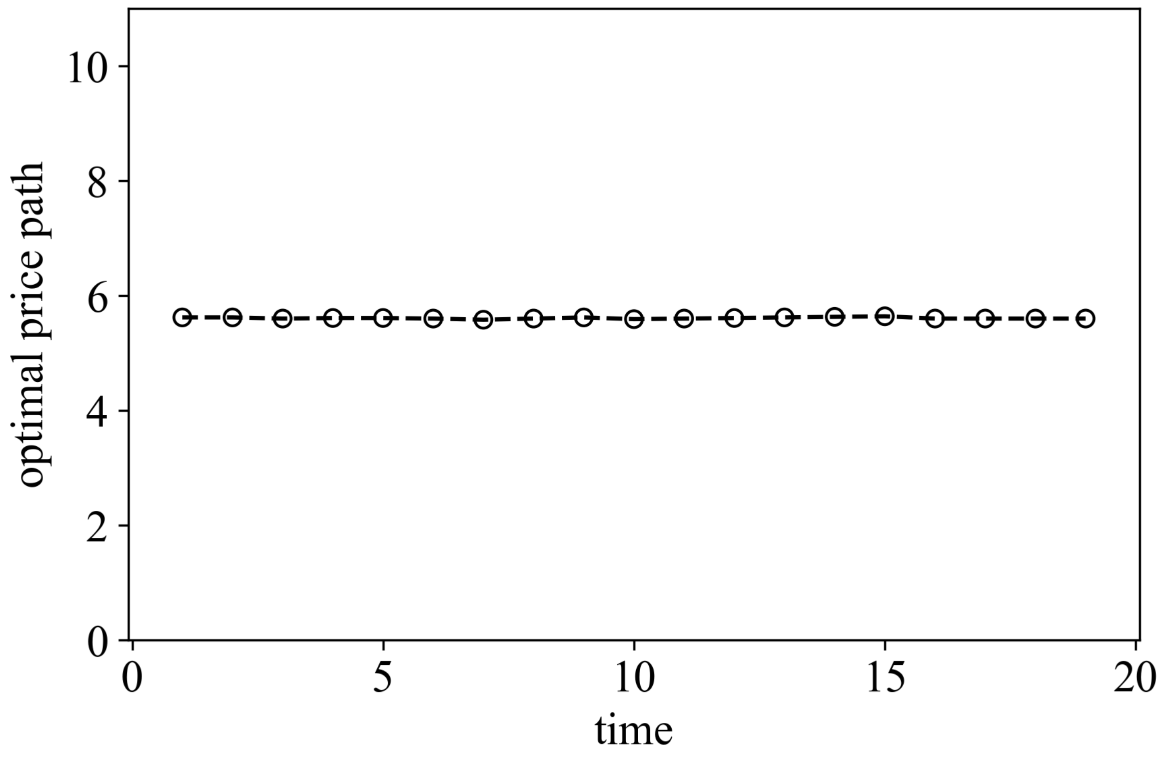

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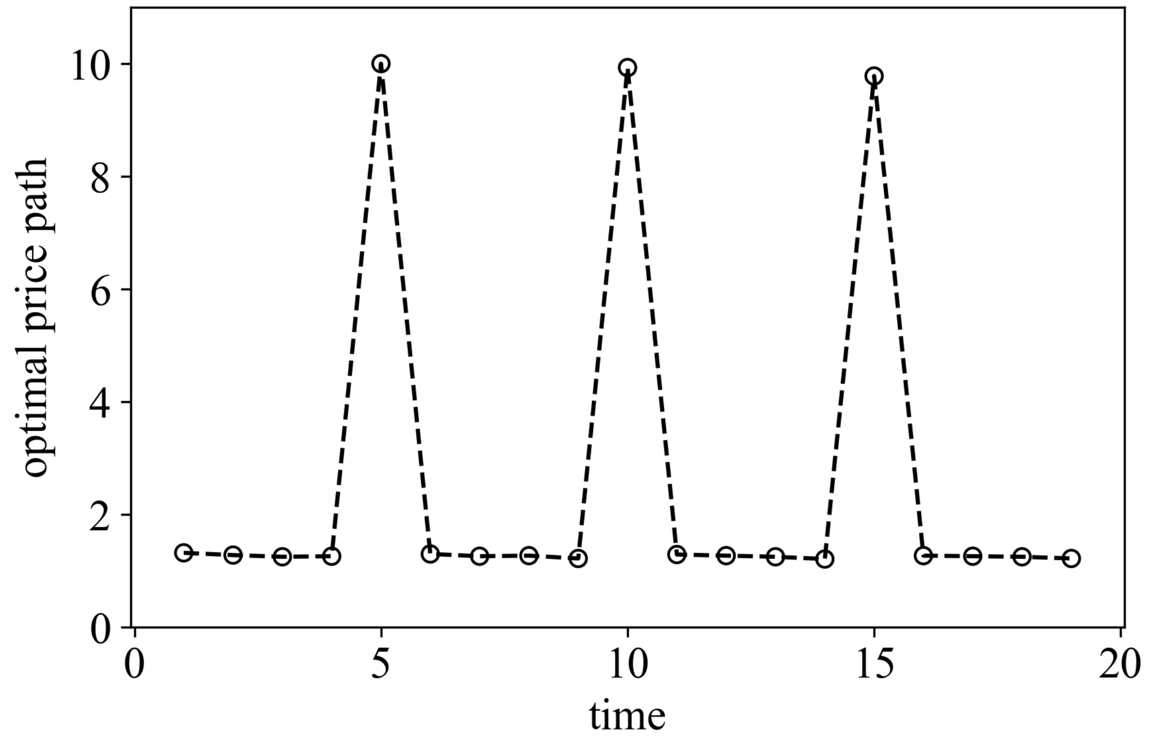
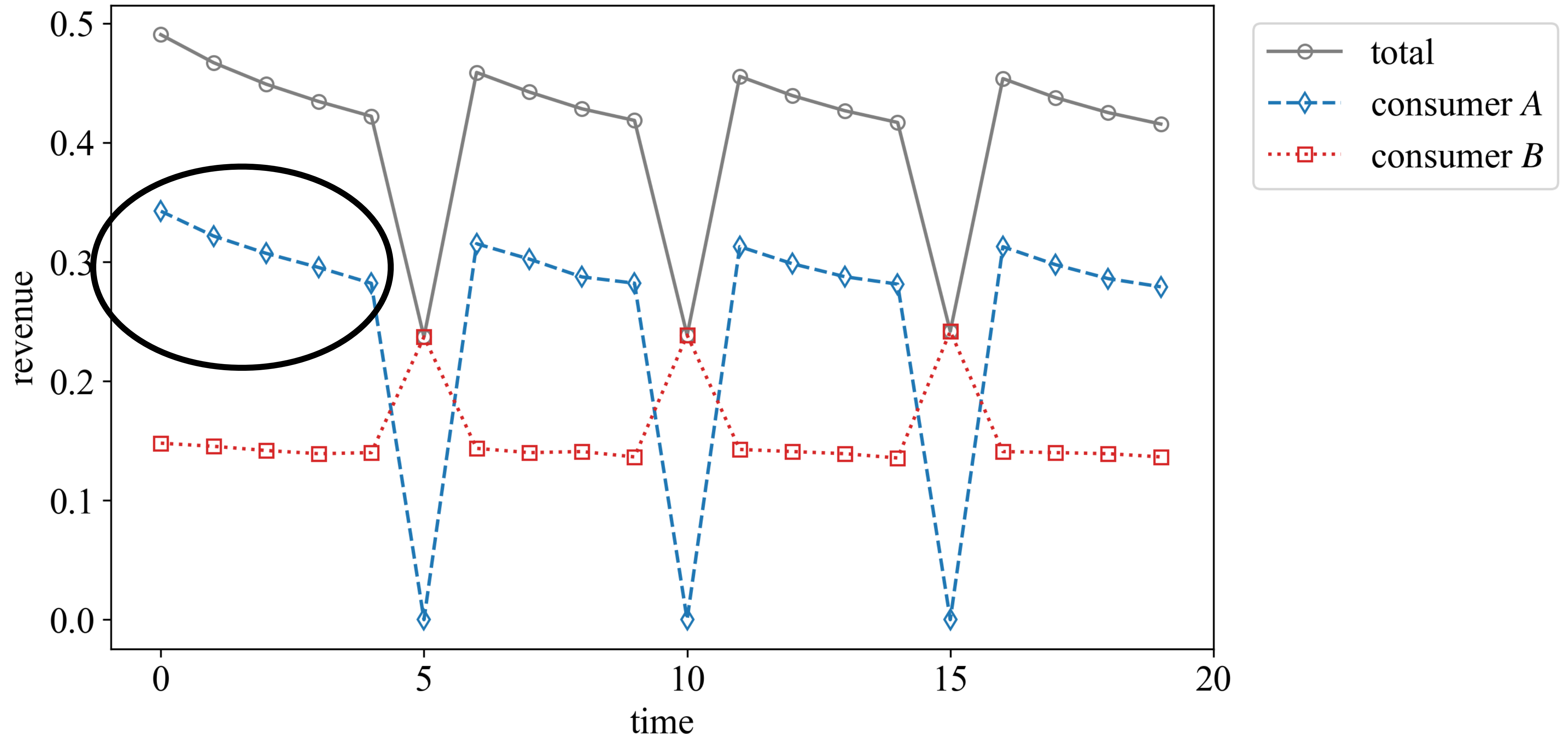


(d) Per period revenue



Example: Two Market Segments

Constant optimal pricing + constant optimal pricing $\neq$ constant optimal pricing

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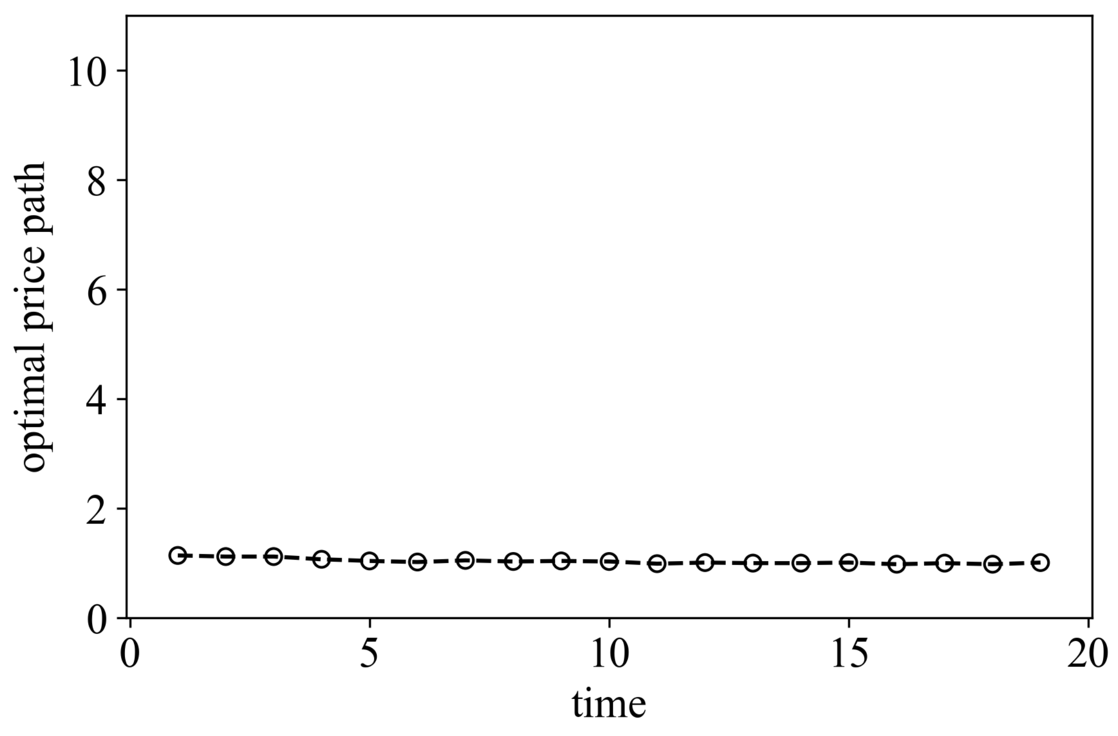

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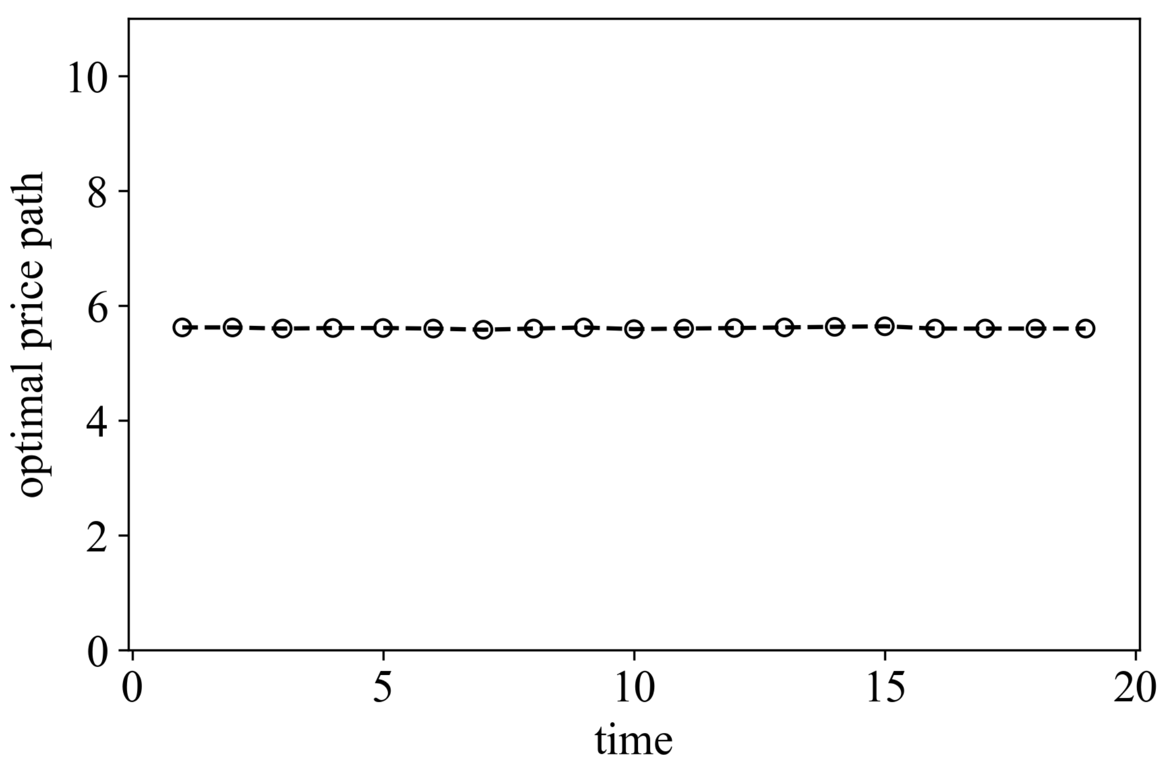

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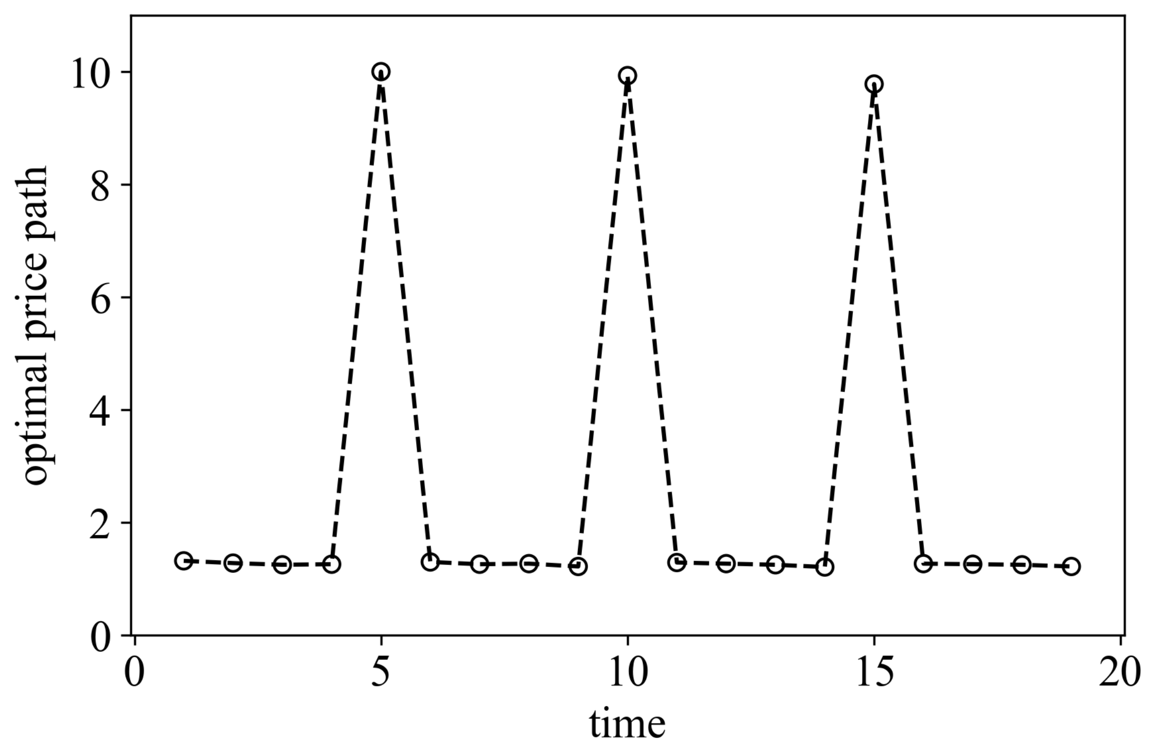

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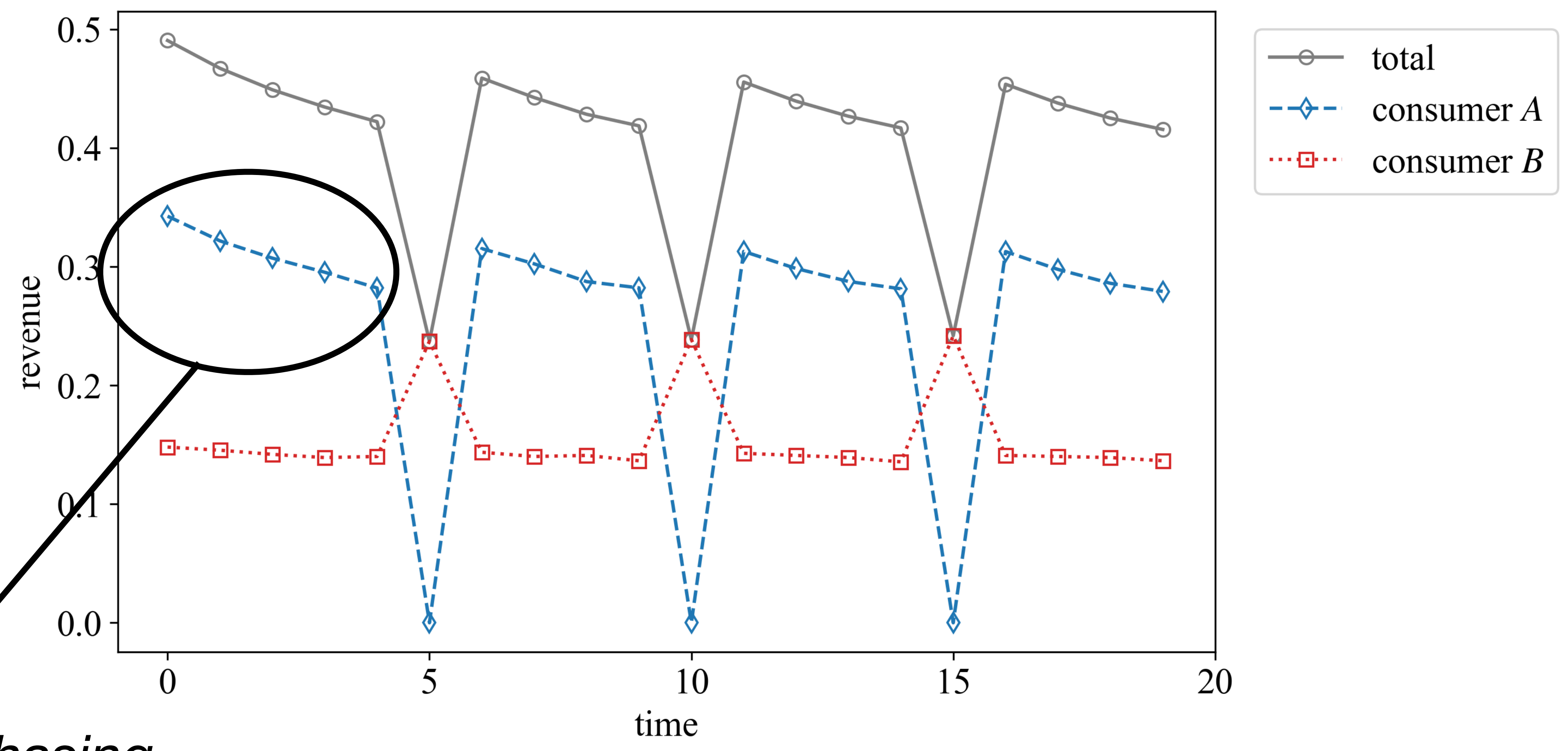

Consumer A mainly purchasing

Example: Two Market Segments

Constant optimal pricing + constant optimal pricing ≠ constant optimal pricing

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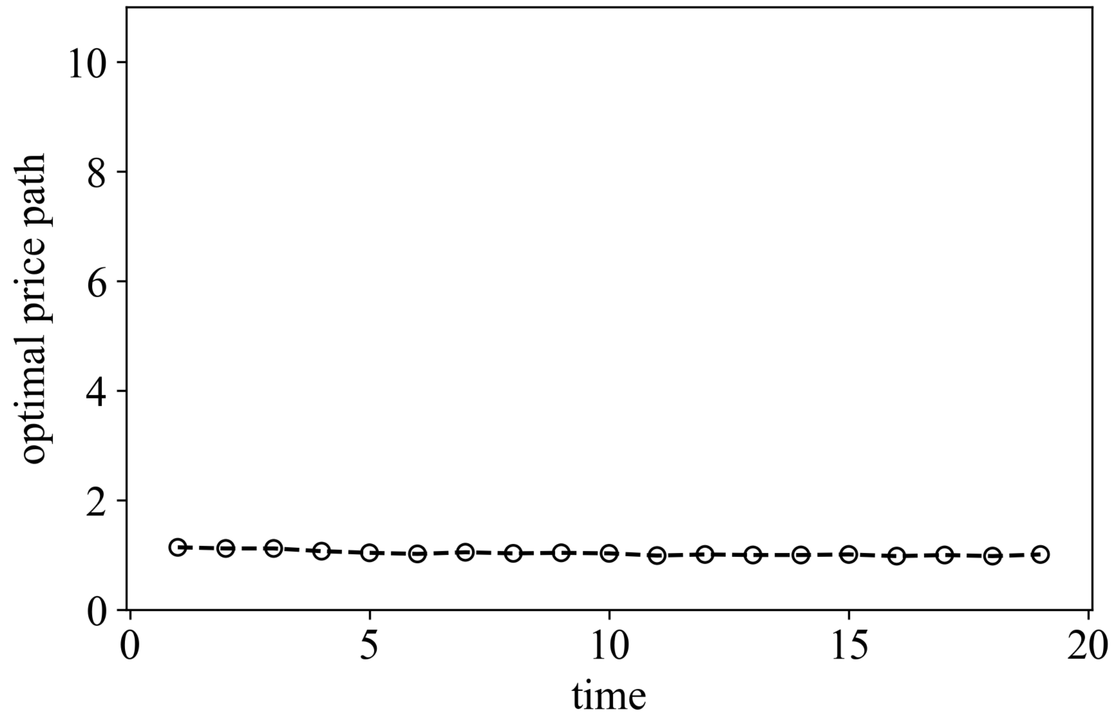

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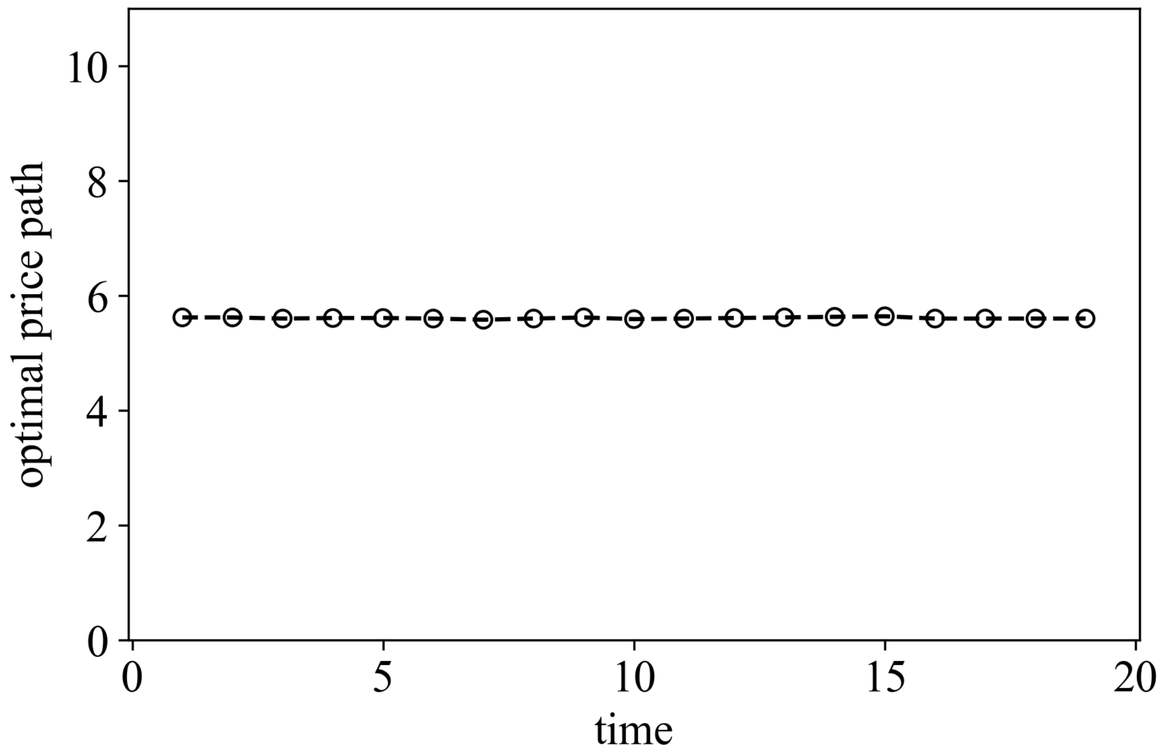

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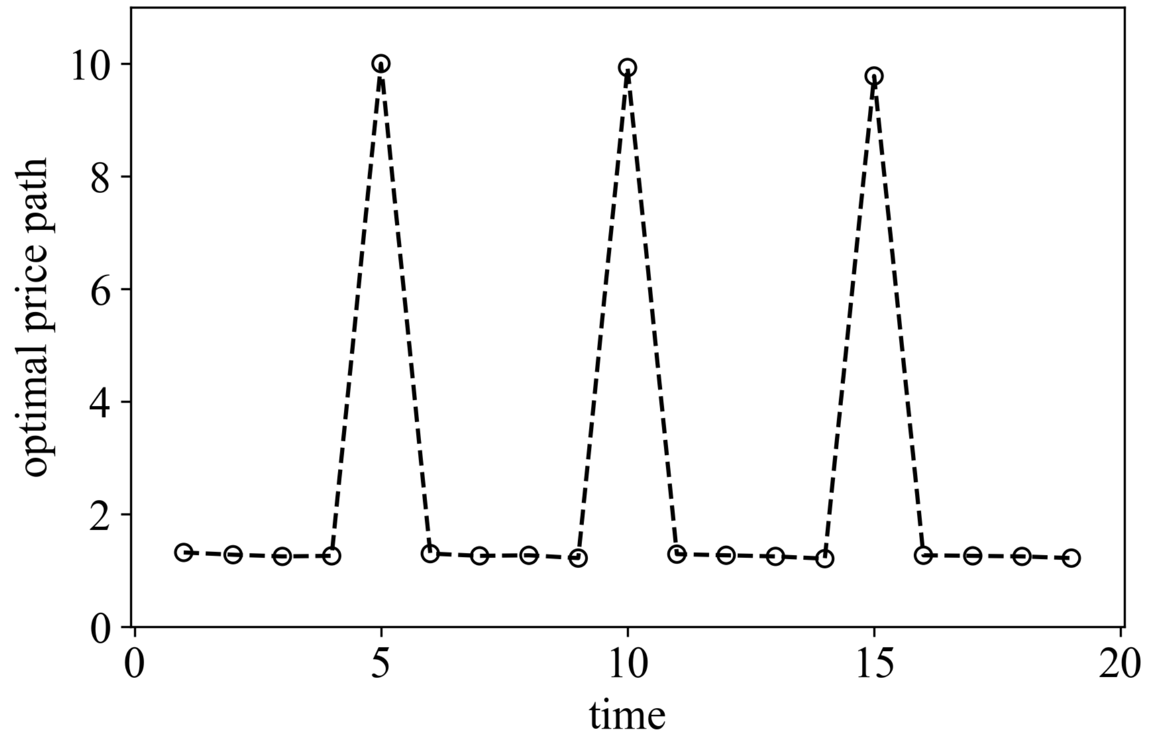

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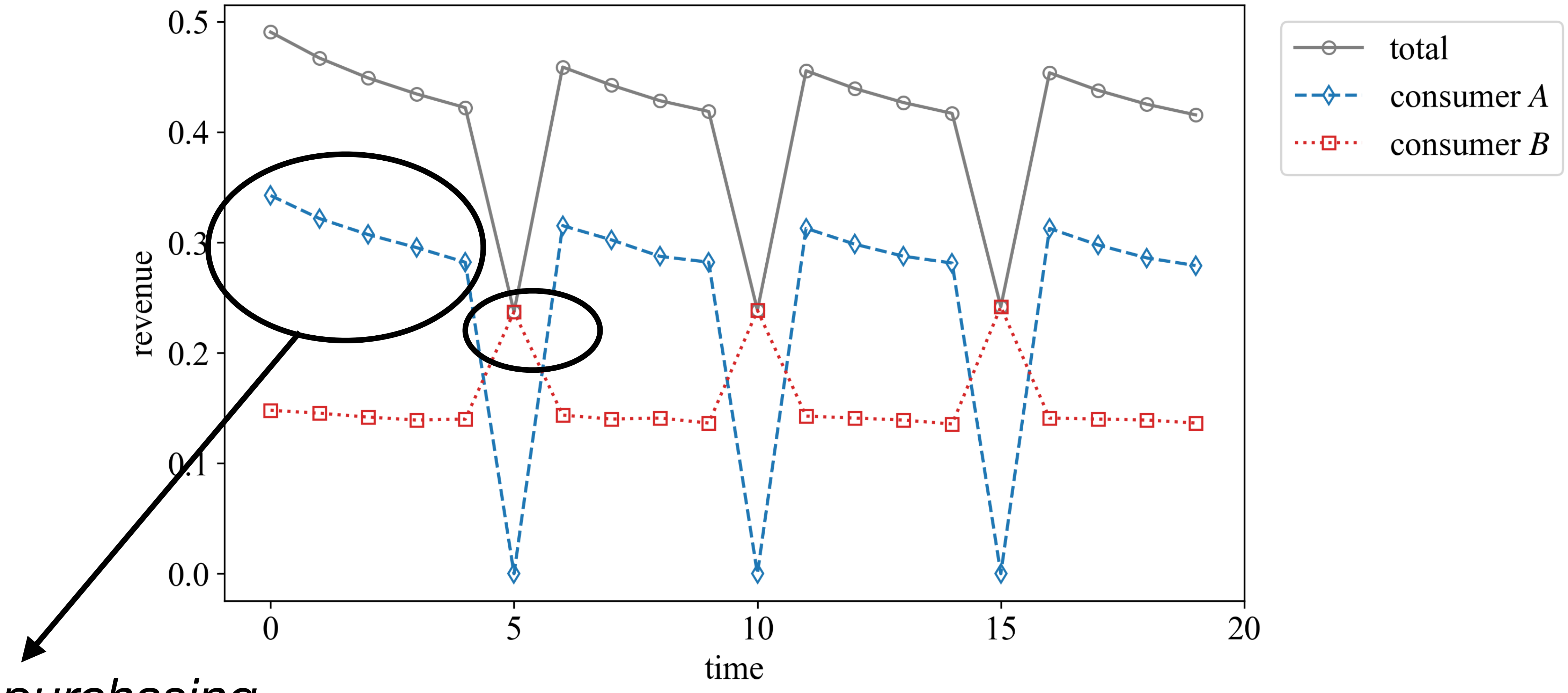

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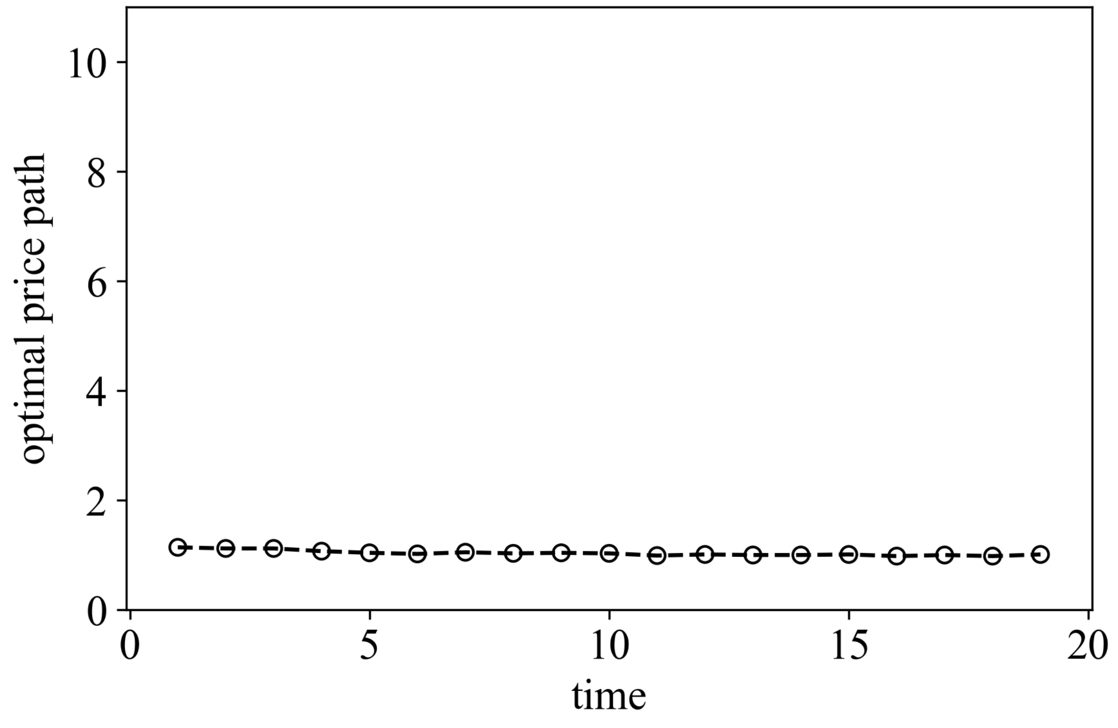

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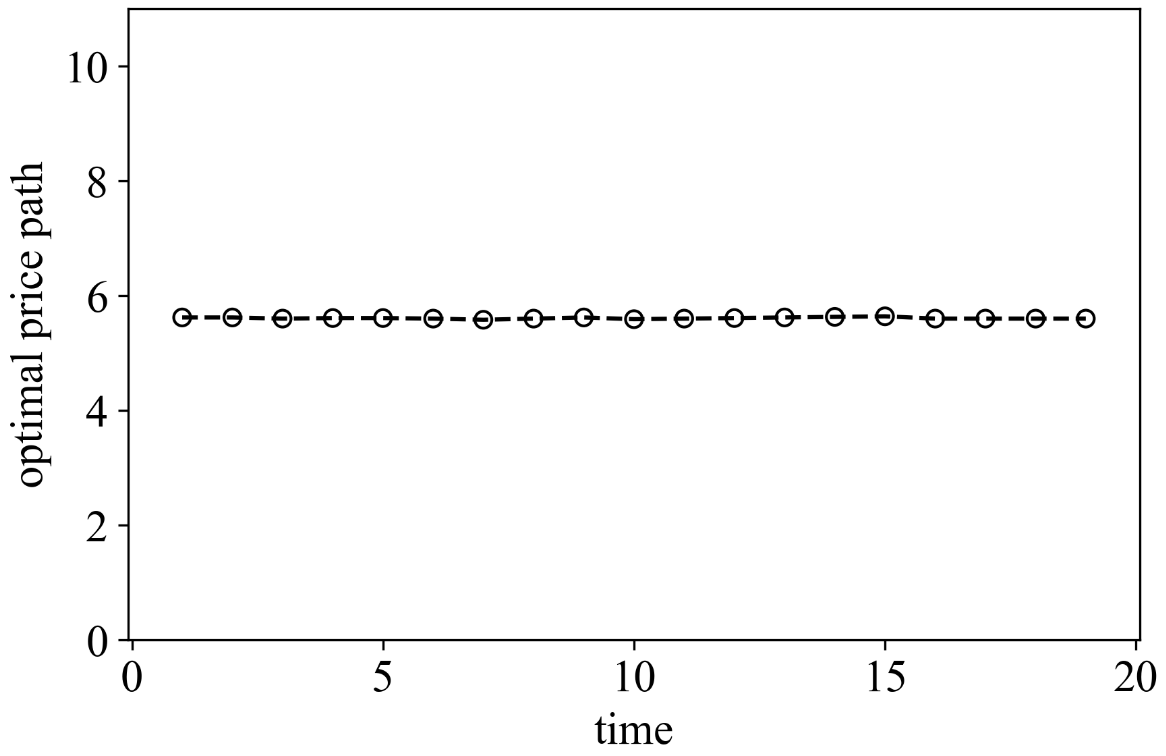

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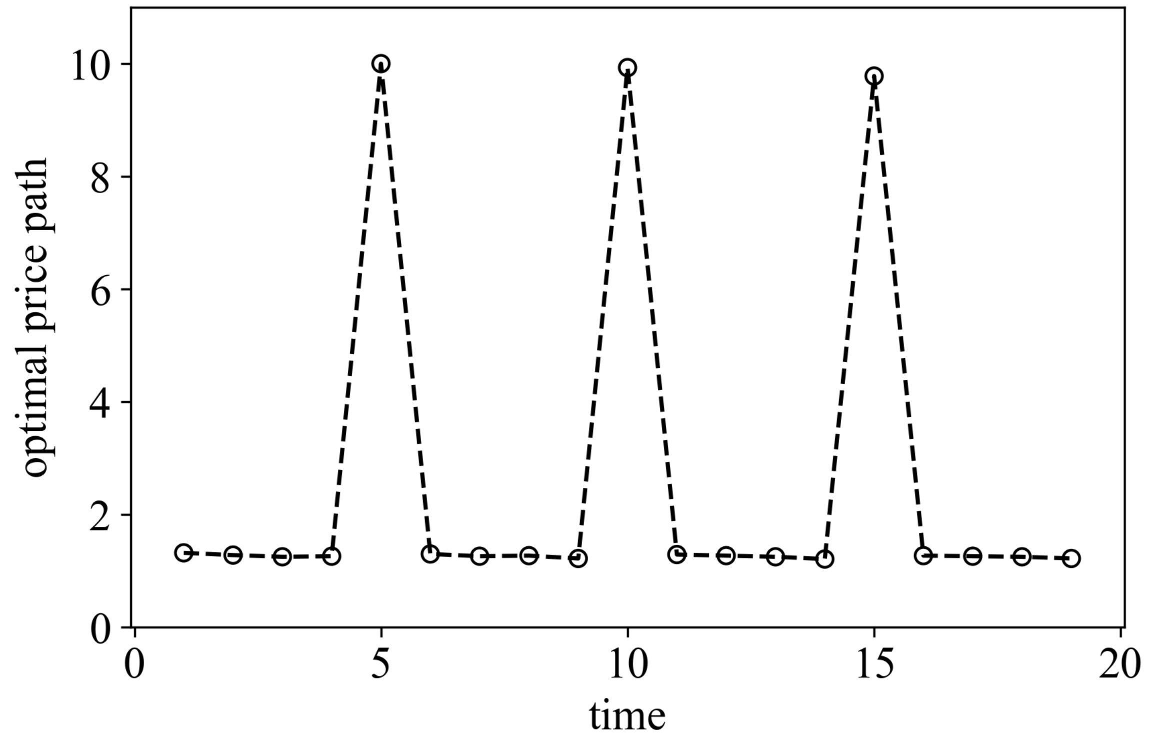

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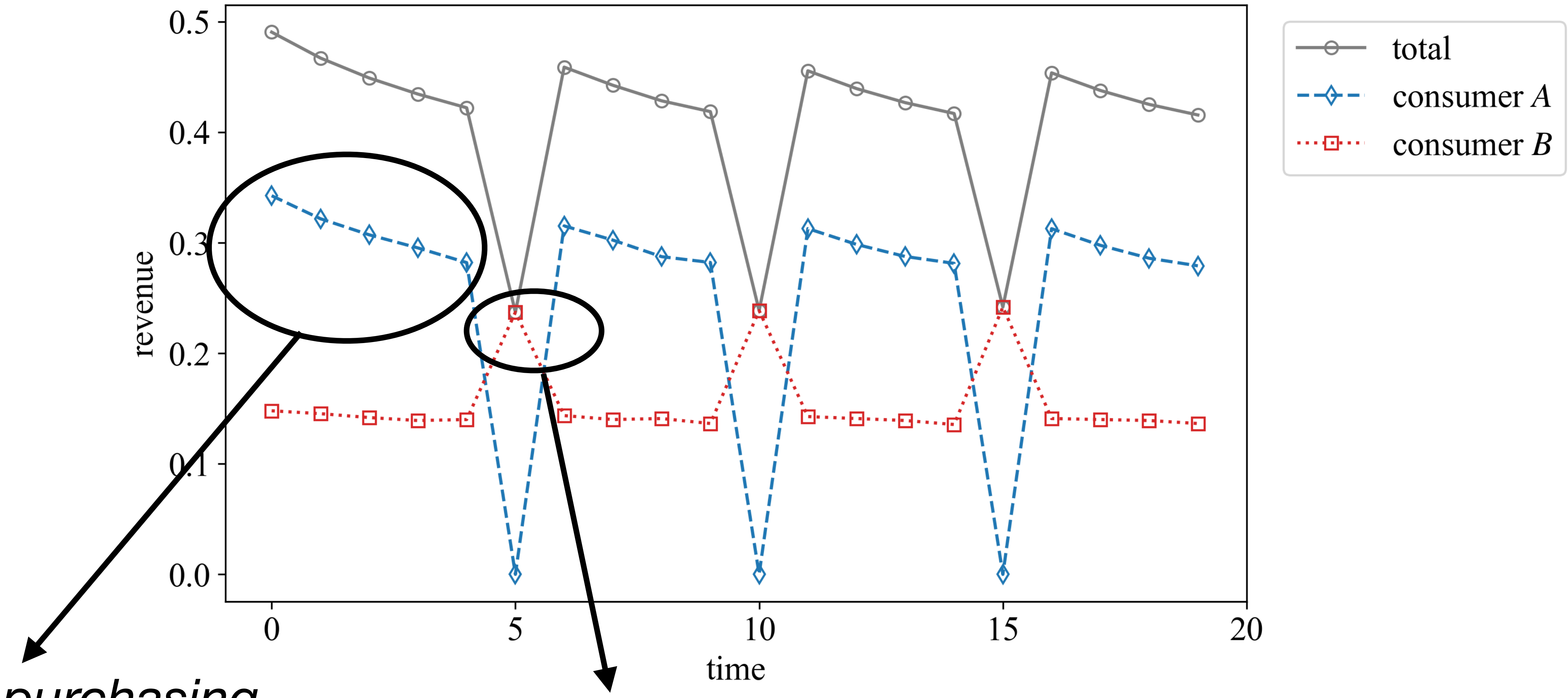

Consumer A mainly purchasing

Consumer B mainly purchasing

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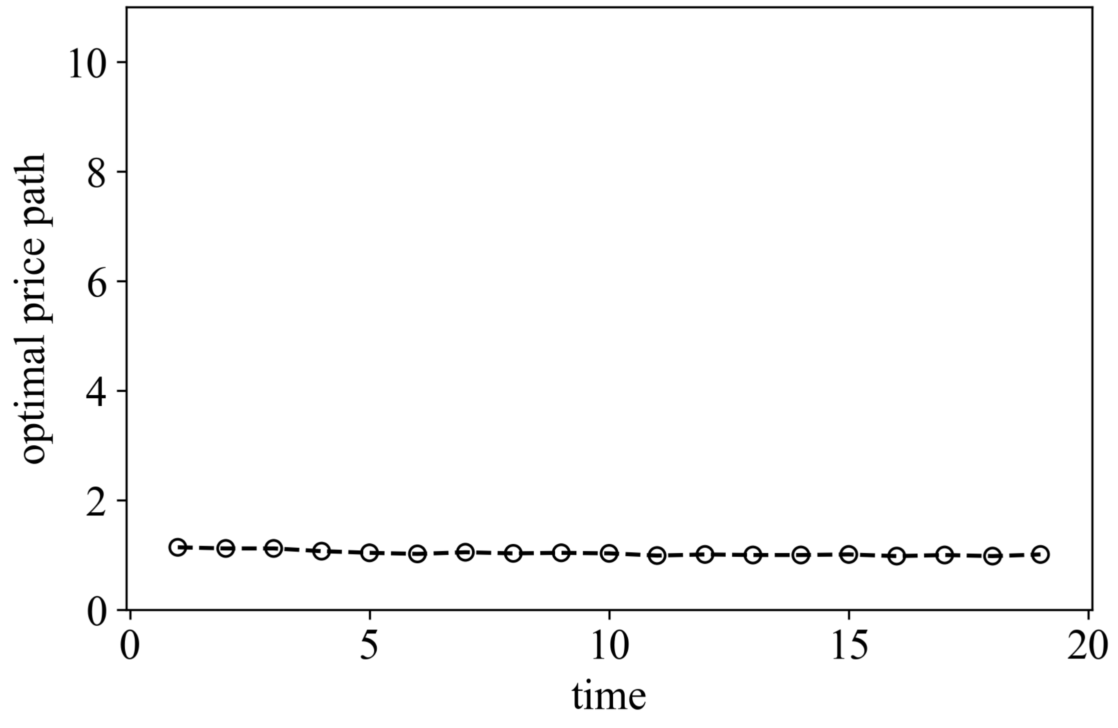

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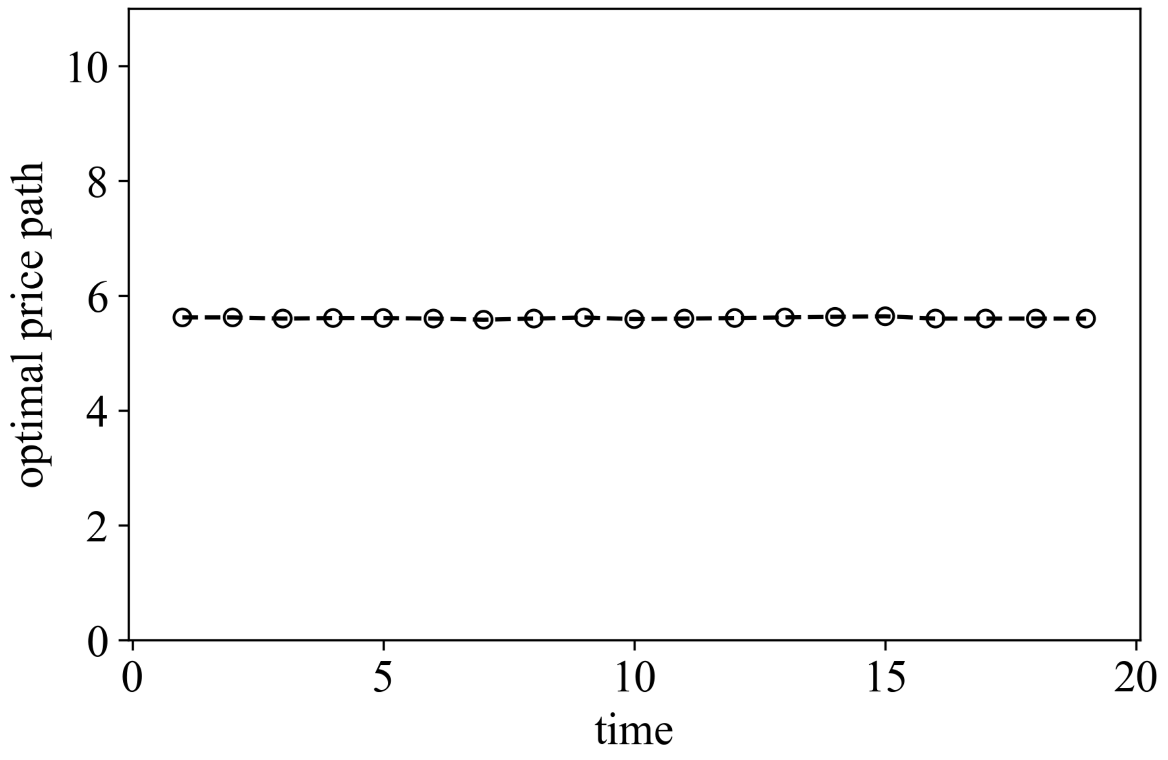

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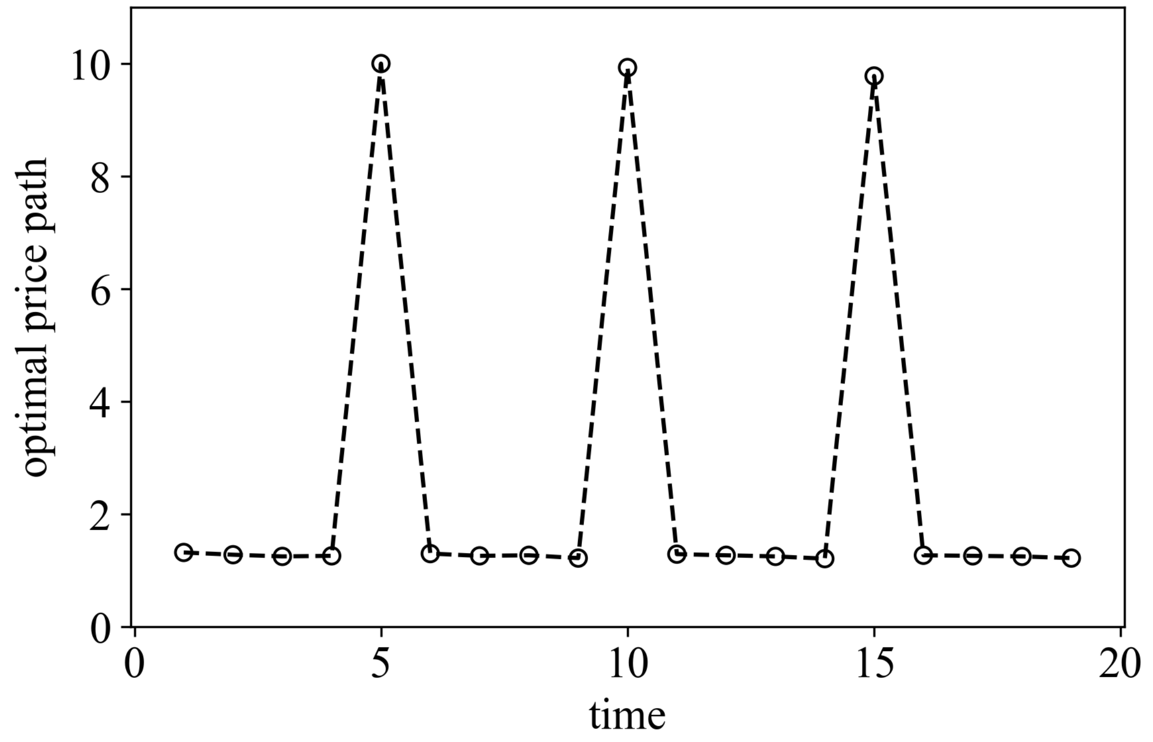

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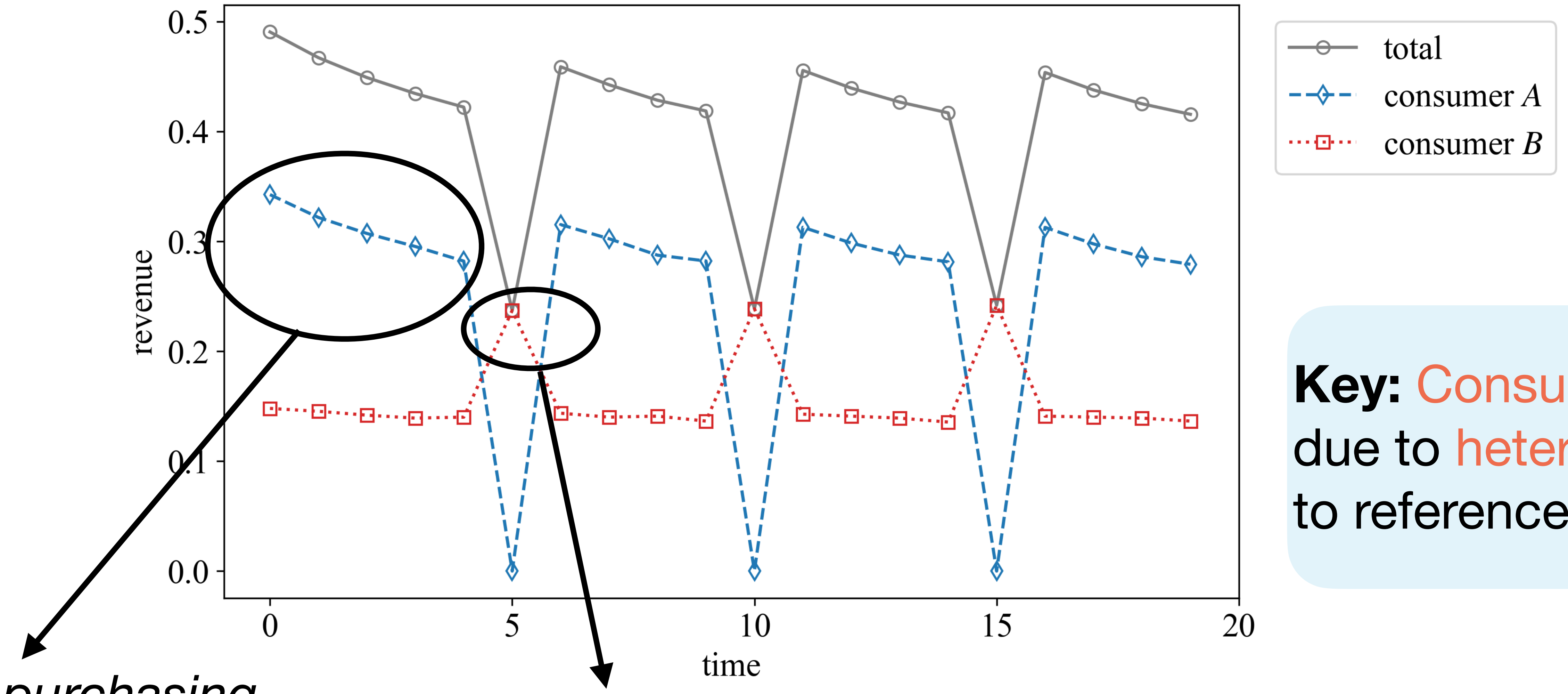


Consumer A mainly purchasing

Consumer B mainly purchasing

Key: Consumer segmentation due to heterogeneous sensitivities to reference price!

Contributions

Formulate the heterogeneous consumer reference effects model in the individual level

Propose a nonparametric statistical method for extracting consumer heterogeneity from transaction data

Provide computational algorithm for optimal pricing policies and establish the sub-optimality of constant policies

Apply to real-world data from retailing platform JD.com and show that the proposed approach leads to significant improvement in revenue

Case Study

Case Study

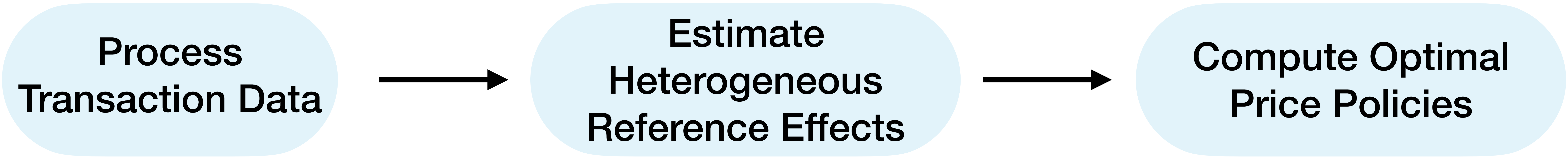
Process Transaction Data

Case Study

JD.COM

Process Transaction Data → Estimate Heterogeneous Reference Effects

Case Study


Process Transaction Data
Estimate Heterogeneous Reference Effects
Compute Optimal Price Policies

Case Study

JD.COM

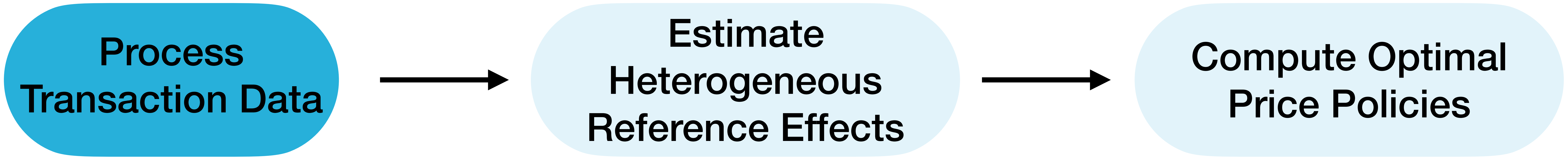
Process Transaction Data → Estimate Heterogeneous Reference Effects → Compute Optimal Price Policies

Case Study JD.COM

Process Transaction Data → Estimate Heterogeneous Reference Effects → Compute Optimal Price Policies

- Transaction data of 30k SKUs (Stock Keeping Unit) from 2.5M consumers

Case Study



- Transaction data of 30k SKUs (Stock Keeping Unit) from 2.5M consumers
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Case Study

JD.COM

Process Transaction Data → Estimate Heterogeneous Reference Effects → Compute Optimal Price Policies

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SKU ID	User ID	Request Time
924eba6741	06102f7920	March 1 23:23

Sample click data in JD.com dataset

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JD.COM

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SKU ID	User ID	Request Time
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Sample click data in JD.com dataset

SKU ID	User ID	Order Time	Selling Price	Original Price
198cec62a	0abe9ef2c	March 1 17:14	79	89

Sample order data in JD.com dataset

Case Study JD.COM

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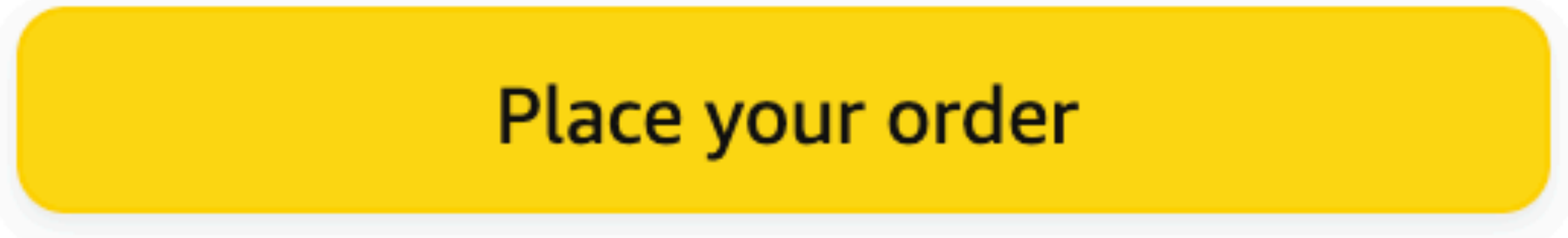

Case Study

JD.COM

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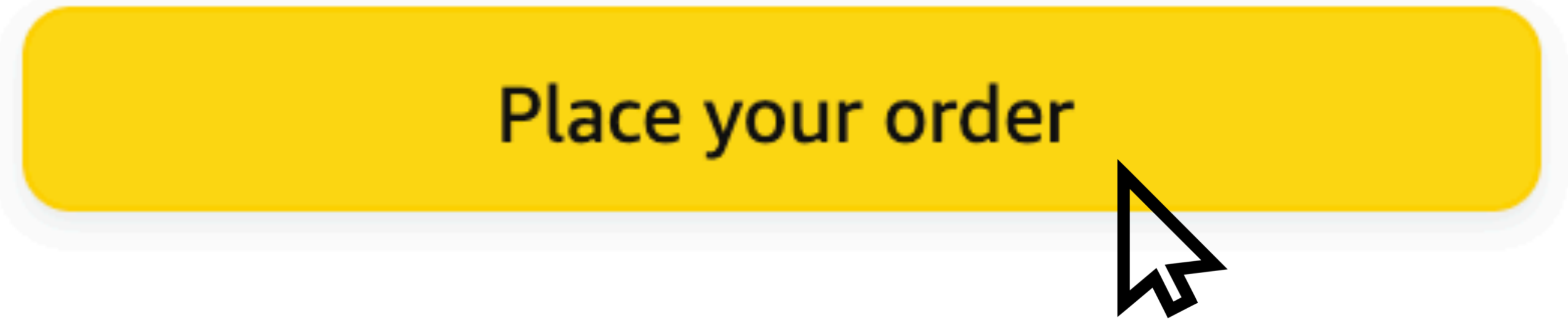

Optimal Price Paths and Implications

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- Focus on most frequently purchased SKUs

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- Focus on most frequently purchased SKUs

Estimated Distribution

Optimal Price Paths and Implications

- Focus on most frequently purchased SKUs

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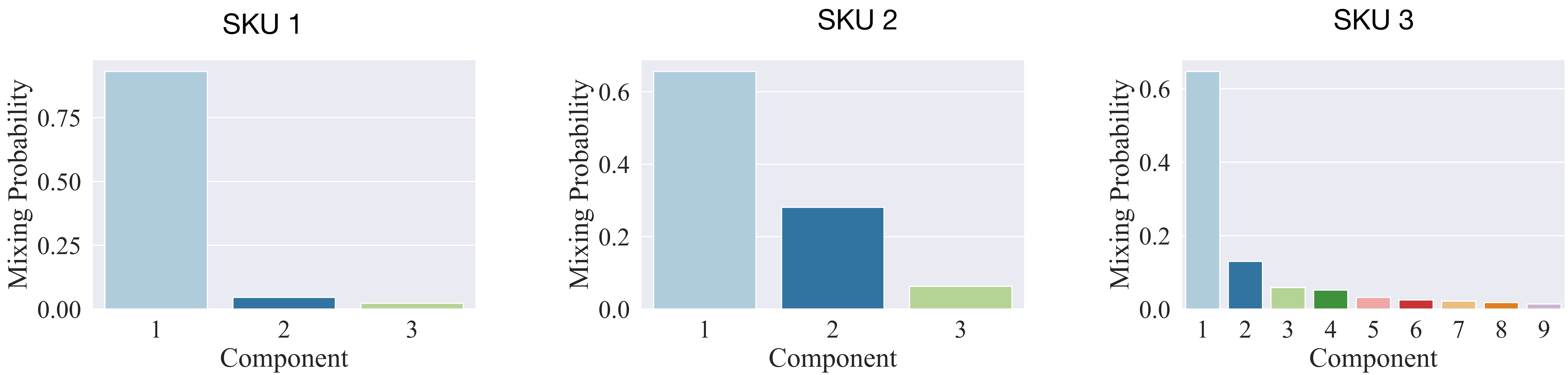
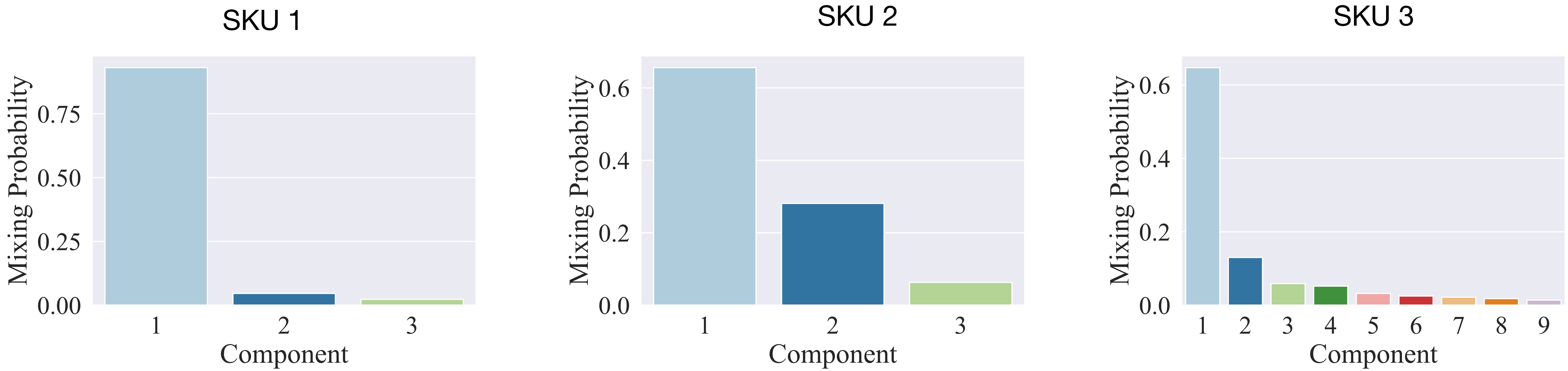


* Only components with mixing probability $\geqslant 0.01$ are shown

Optimal Price Paths and Implications

- Focus on most frequently purchased SKUs

Estimated Distribution



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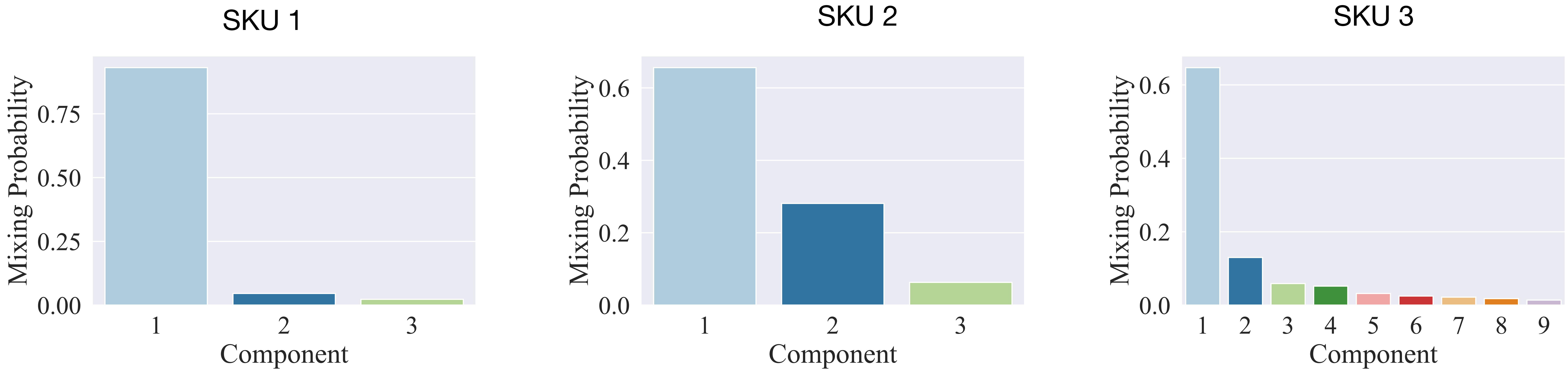
Optimal Price Path

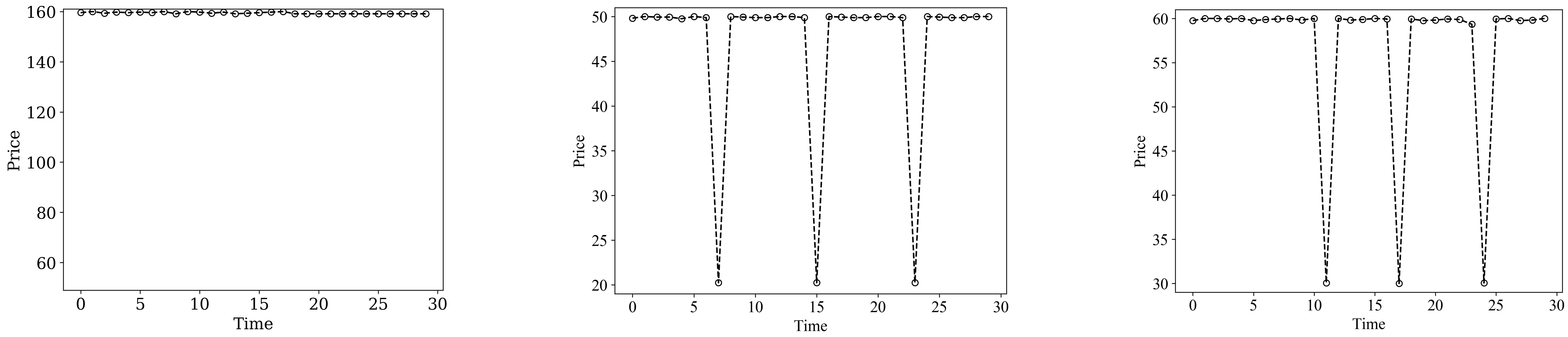
Optimal Price Paths and Implications

- Focus on most frequently purchased SKUs

Estimated Distribution



Optimal Price Path



Numerical Comparisons

Methods

NPMLE: Proposed approach

EM: Finite mixed logit model estimated by Expectation-Maximization

SL: Single logit model

Lin: Piecewise linear model for aggregate level data

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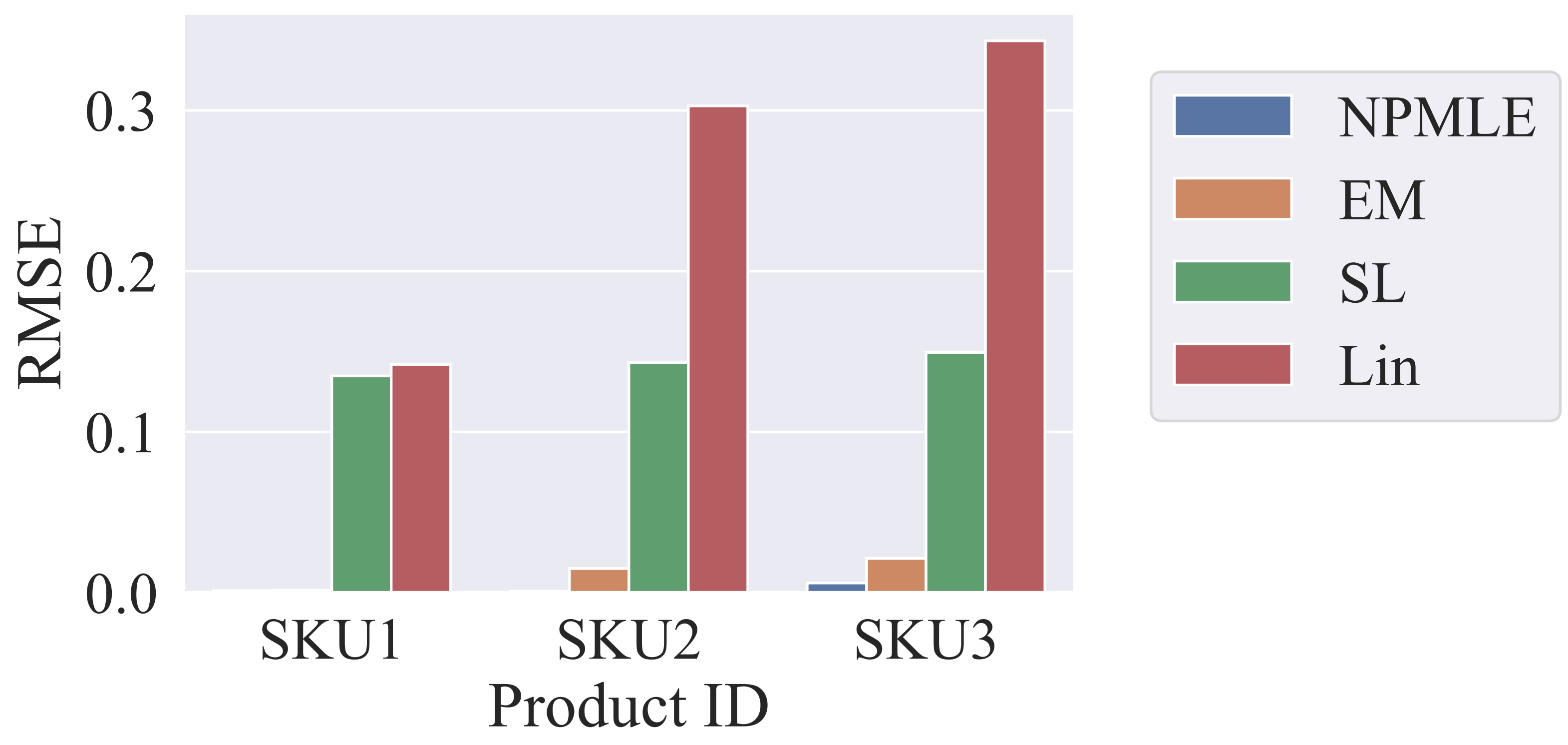

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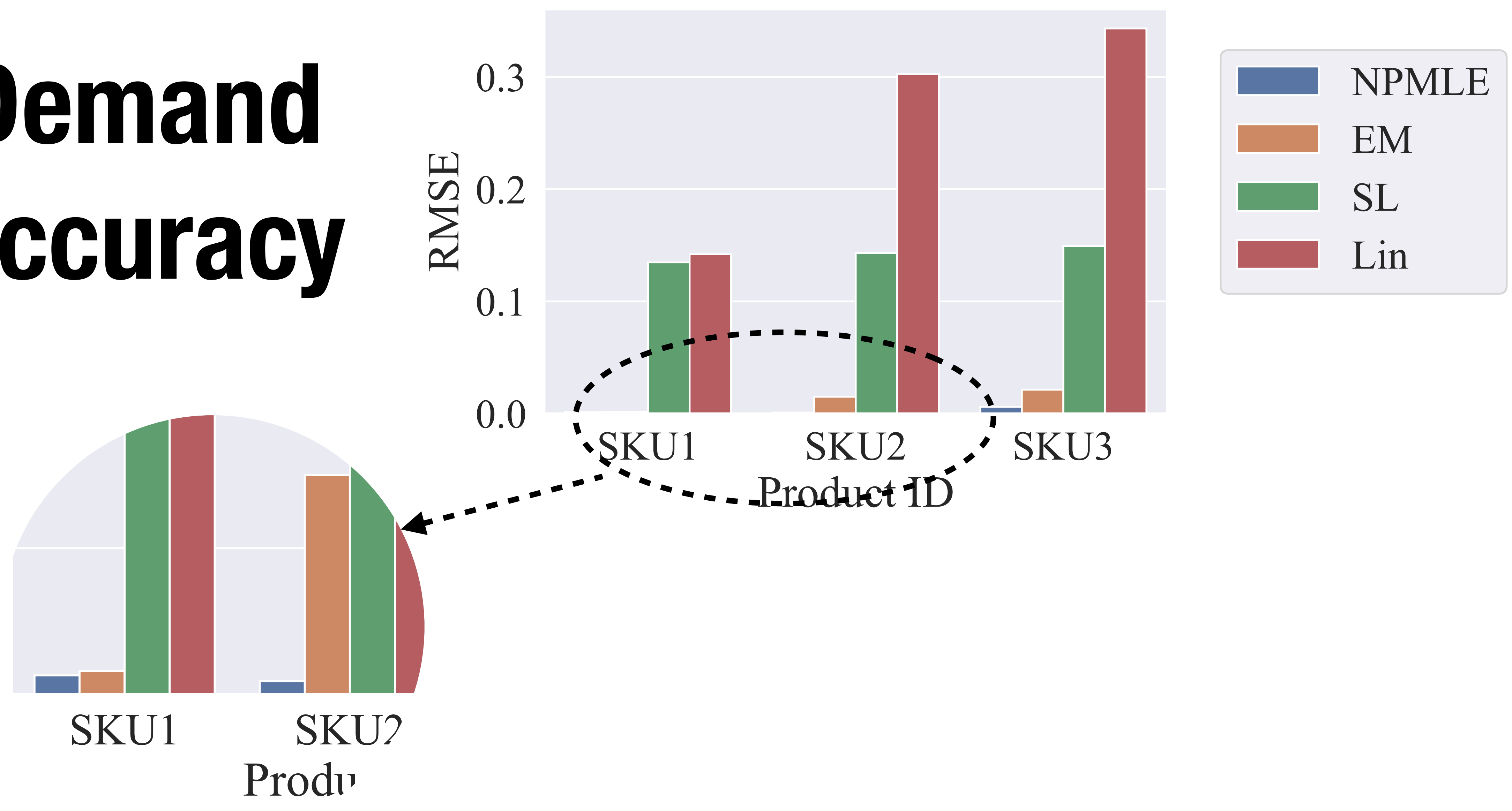

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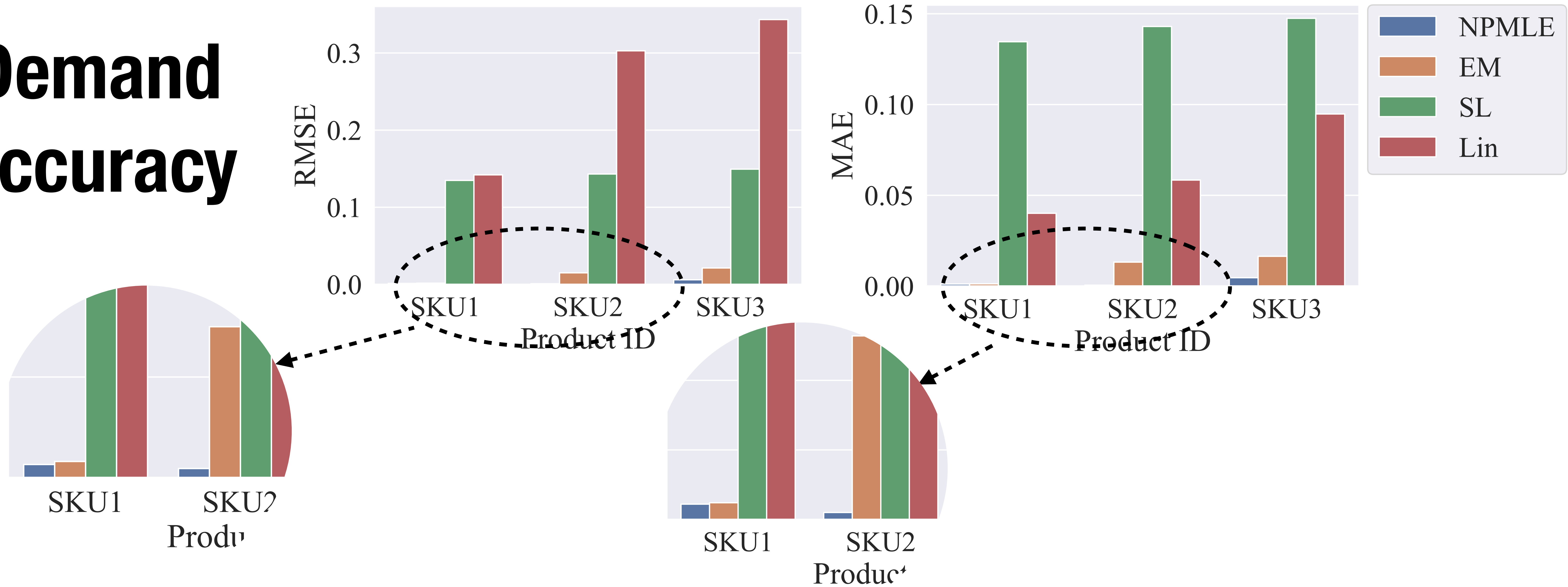

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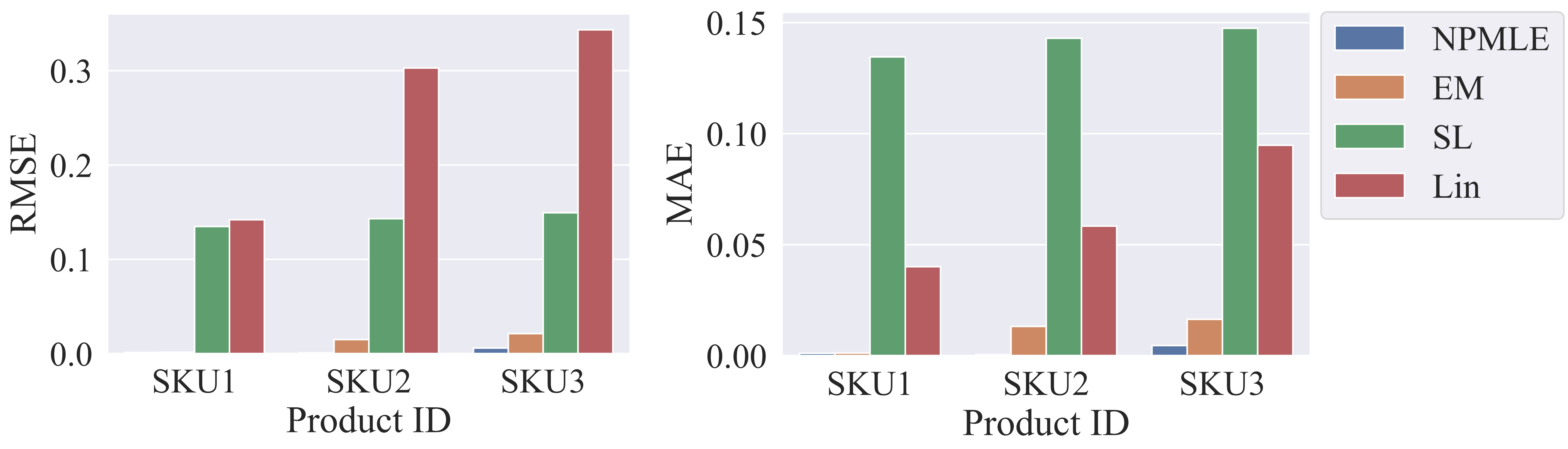

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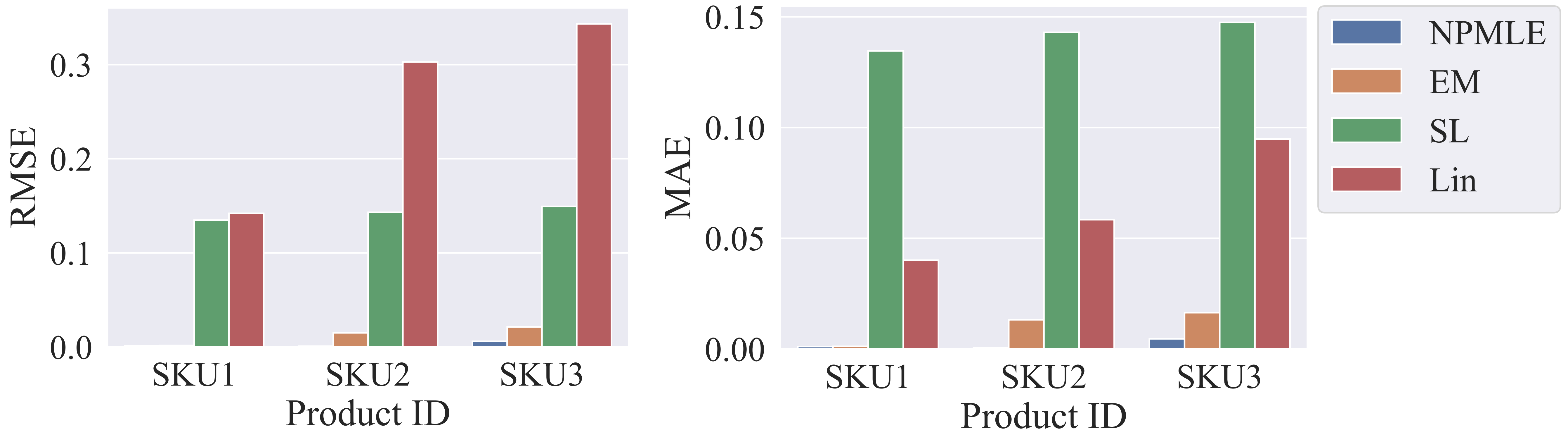

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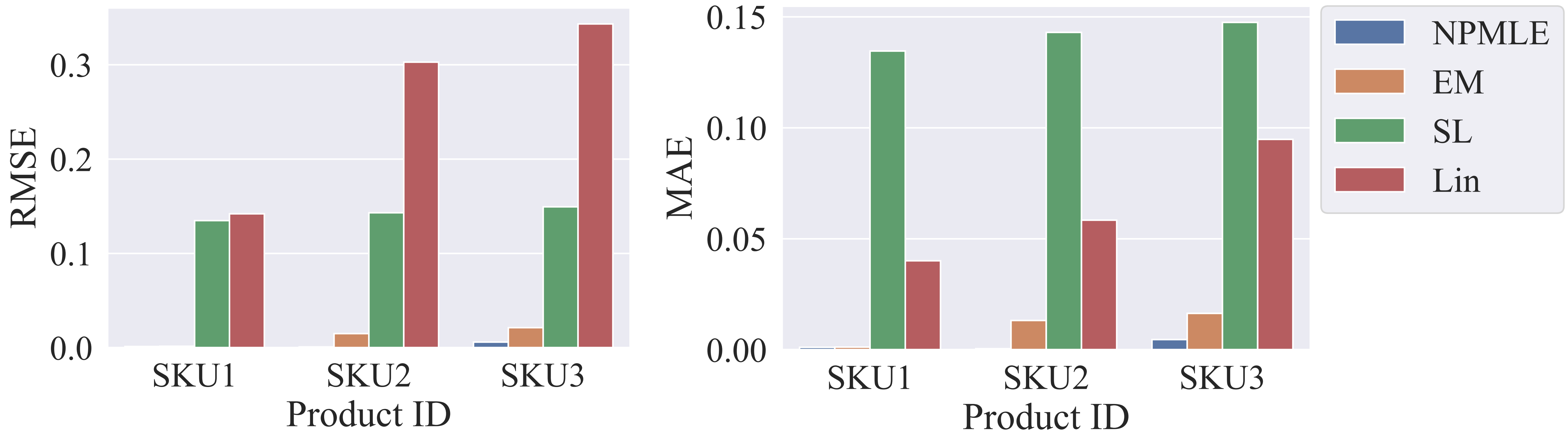

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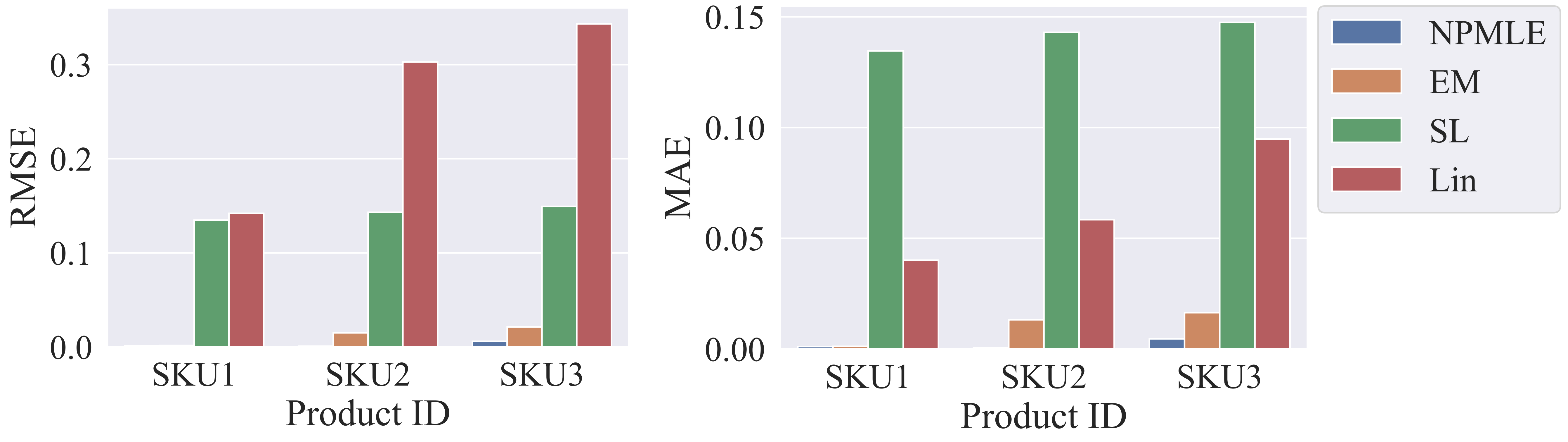

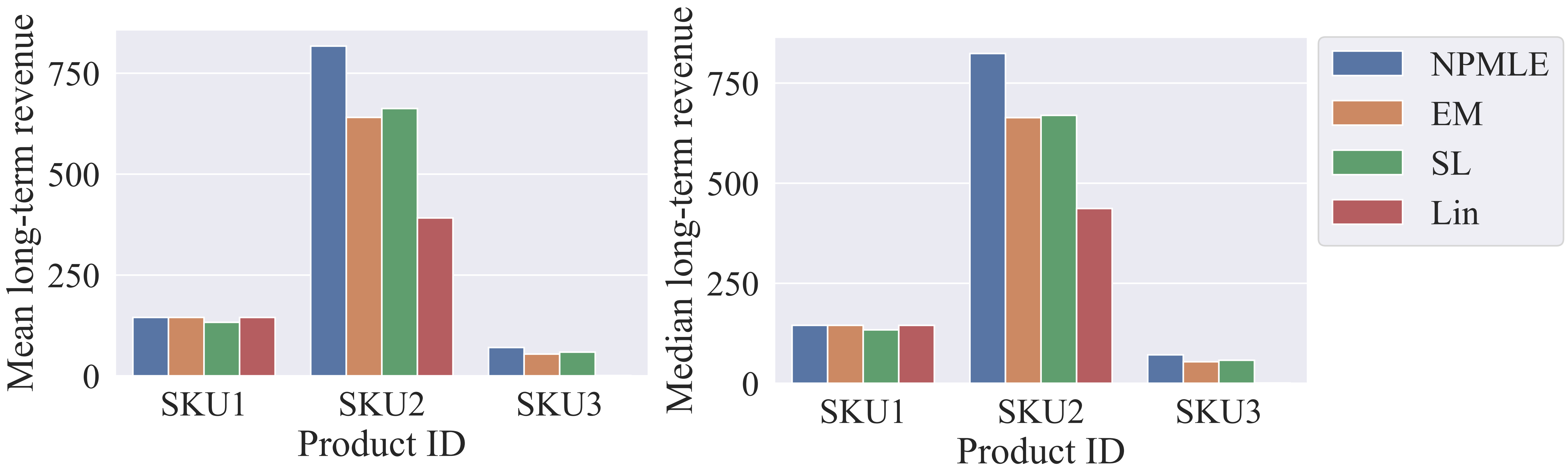

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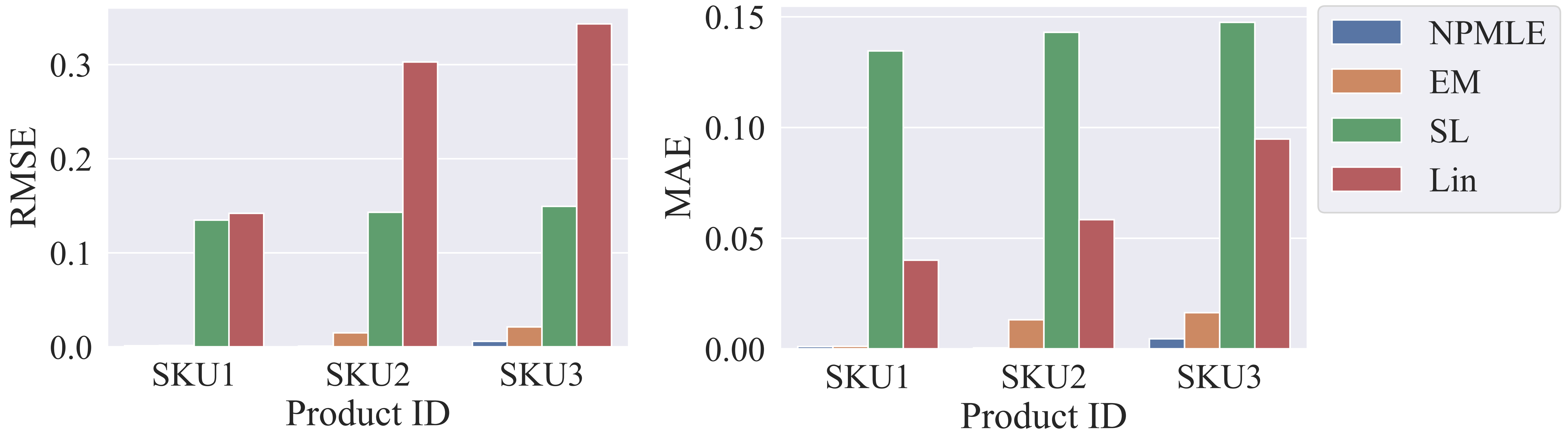

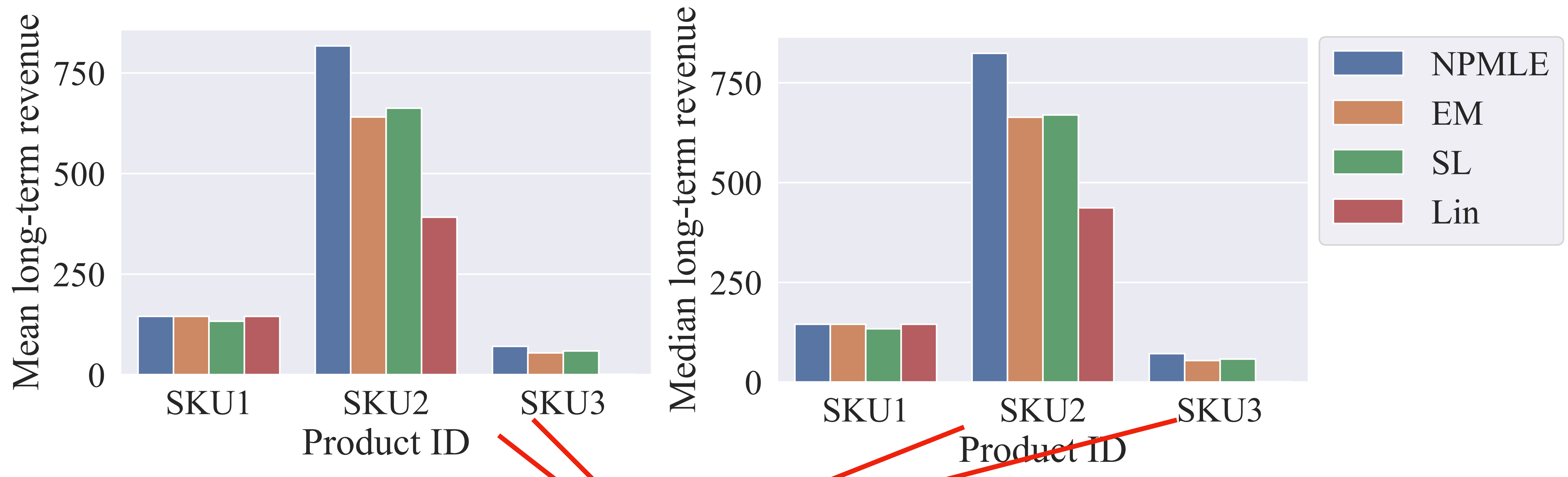

Average $\geqslant$ 30% increase in revenue!

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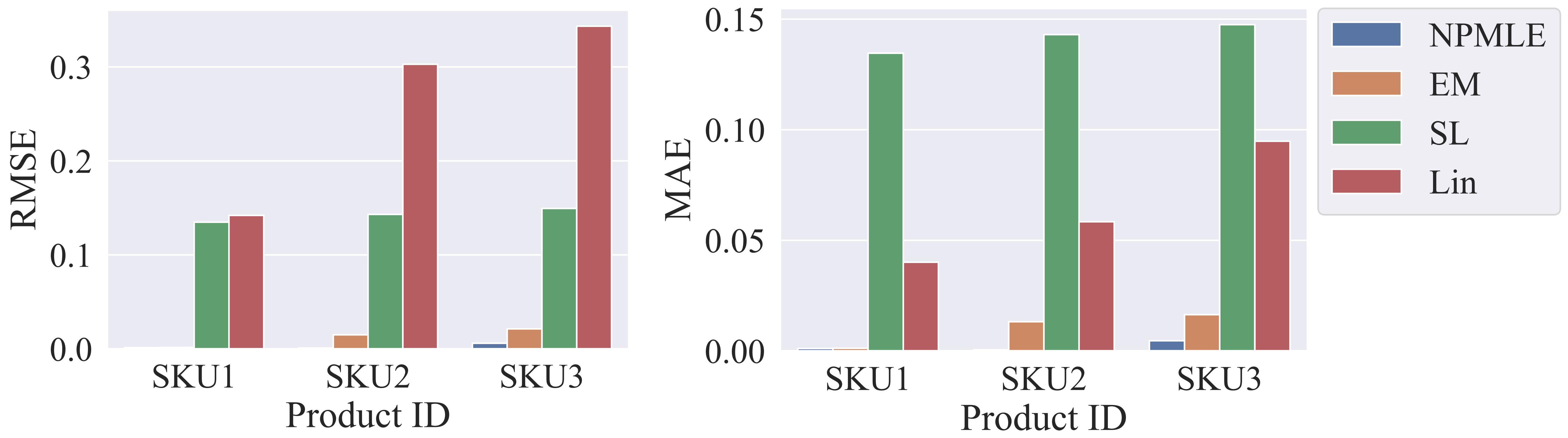

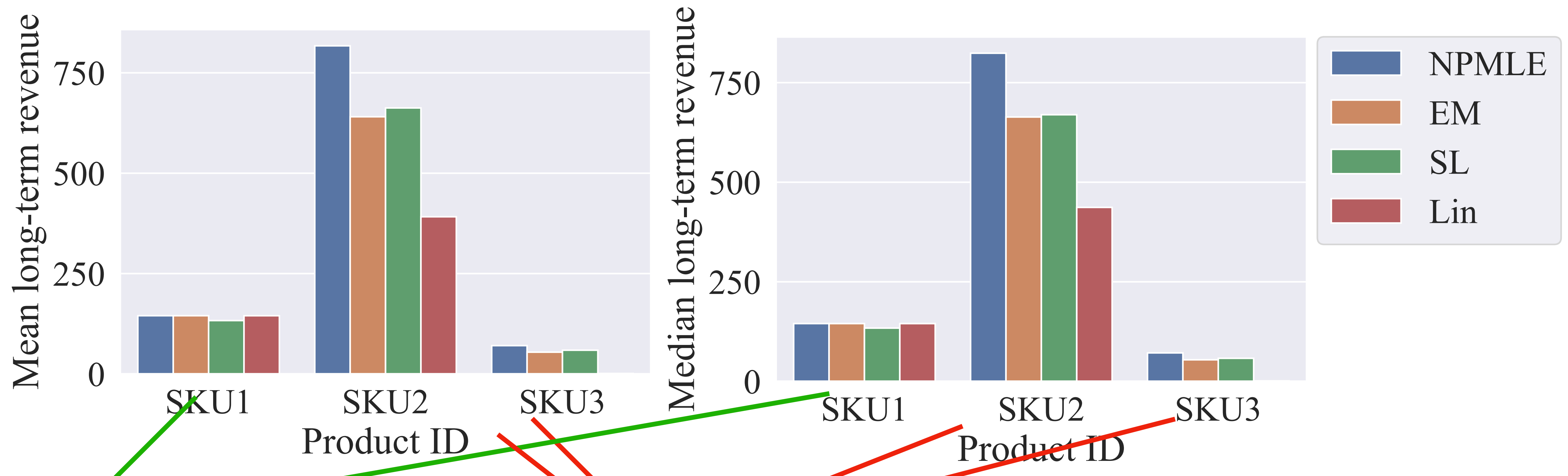

Less revenue increase when constant policy is optimal

Average ⩾ 30% increase in revenue!

Summary

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Takeaway Message

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- Field experiments and practical impact
- More general reference price model
  - Horizontal reference effect in a multi-product setting
  - Stochastic updating scheme and estimation

Extension to Multi-Product Setting

"Multi-Product Dynamic Pricing with Reference Effects Under Logit Demand". Under 2nd-round review at *Operations Research.* Amy Guo, **H. Jiang**, Z.-J. Max Shen.

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Theorem (informal)
Suppose the reference effects of all products are gain-seeking, then the optimal pricing policy admits no steady state.

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Theorem (informal)
The optimal steady state price, if exists, admits an explicit characterization depending on sensitivity parameters, memory parameter, and discount factor, and the steady state price can be computed efficiently.

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Thanks for your attention! Questions?

Supplementary slides

Explicit Characterization of Optimal Steady State

Theorem Consider loss-neutral case with N products. If the optimal pricing policy admits a steady state such that $\mathbf{p}^\star(\mathbf{p}^{\star\star}) = \mathbf{p}^{\star\star}$, then $\mathbf{p}^{\star\star}$ satisfies

$$p_i^{\star\star} = \Pi^{\star\star} + \frac{1}{b_i + c_i\kappa}, \quad \forall i \in N,$$

where $\kappa := (1-\beta)/(1-\alpha\beta)$, and $\Pi^{\star\star}$ is the single-period revenue at the optimal steady state, which is the unique solution to the equation

$$\Pi = \sum_{i\in N} \frac{1}{b_i + c_i\kappa} \cdot \exp\left(a_i - b_i\Pi - \frac{b_i}{b_i + c_i\kappa}\right).$$

Implications

- Optimal prices of different products differ based on b_i and c_i
- Efficient computation of optimal prices by binary search

Sub-optimality Results

Theorem (Sub-optimality of constant pricing policy, informal)
For sufficiently large c_-, the constant pricing policy is **not** optimal even if $c_+ \leq c_-$ (loss-averse or neutral) .

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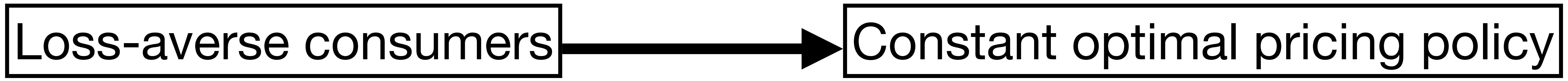

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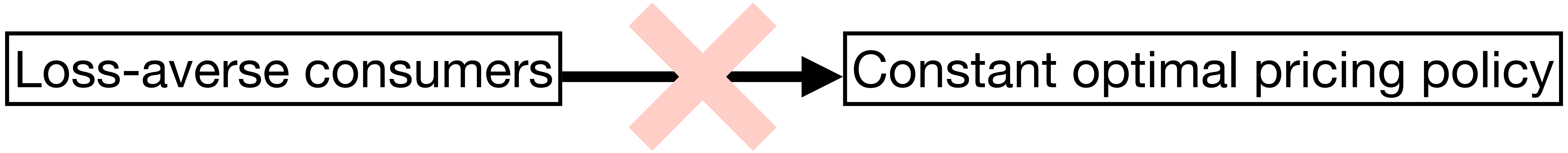

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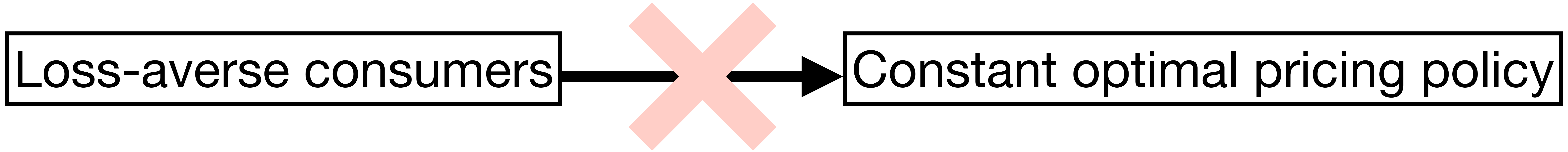

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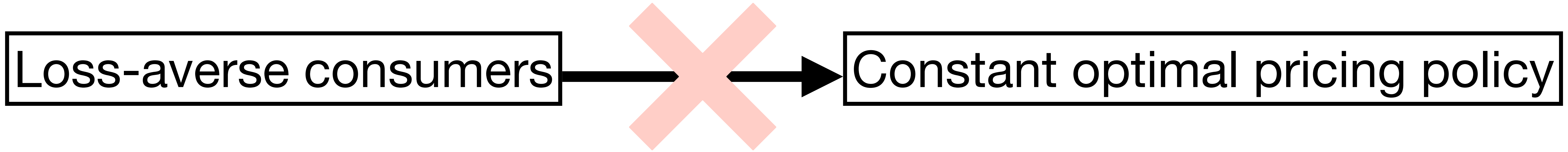

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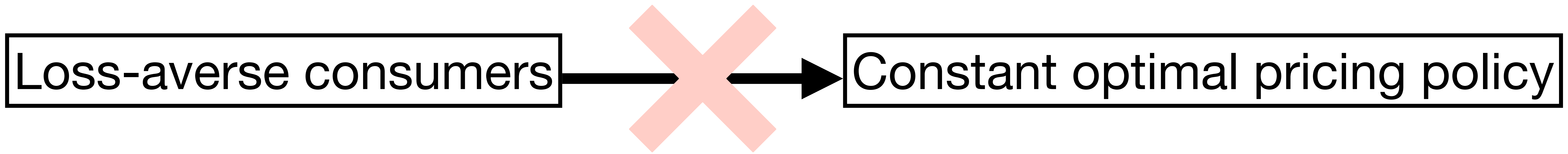

More general sub-optimality results than existing works

- **Removes** simplified assumption that memory parameter $\alpha = 0$ [Z. Hu, J. Nasiry (2017)]
- Holds for individual level model with ***arbitrary*** number of segments rather than only two segments [N. Chen, J. Nasiry (2020)]

Illustrations of Demand Model

$x = r - p$
$R(x,p) = \text{Demand}(x+p,p) - \text{Demand}(p,p)$

“Decreasing Curvature” Property

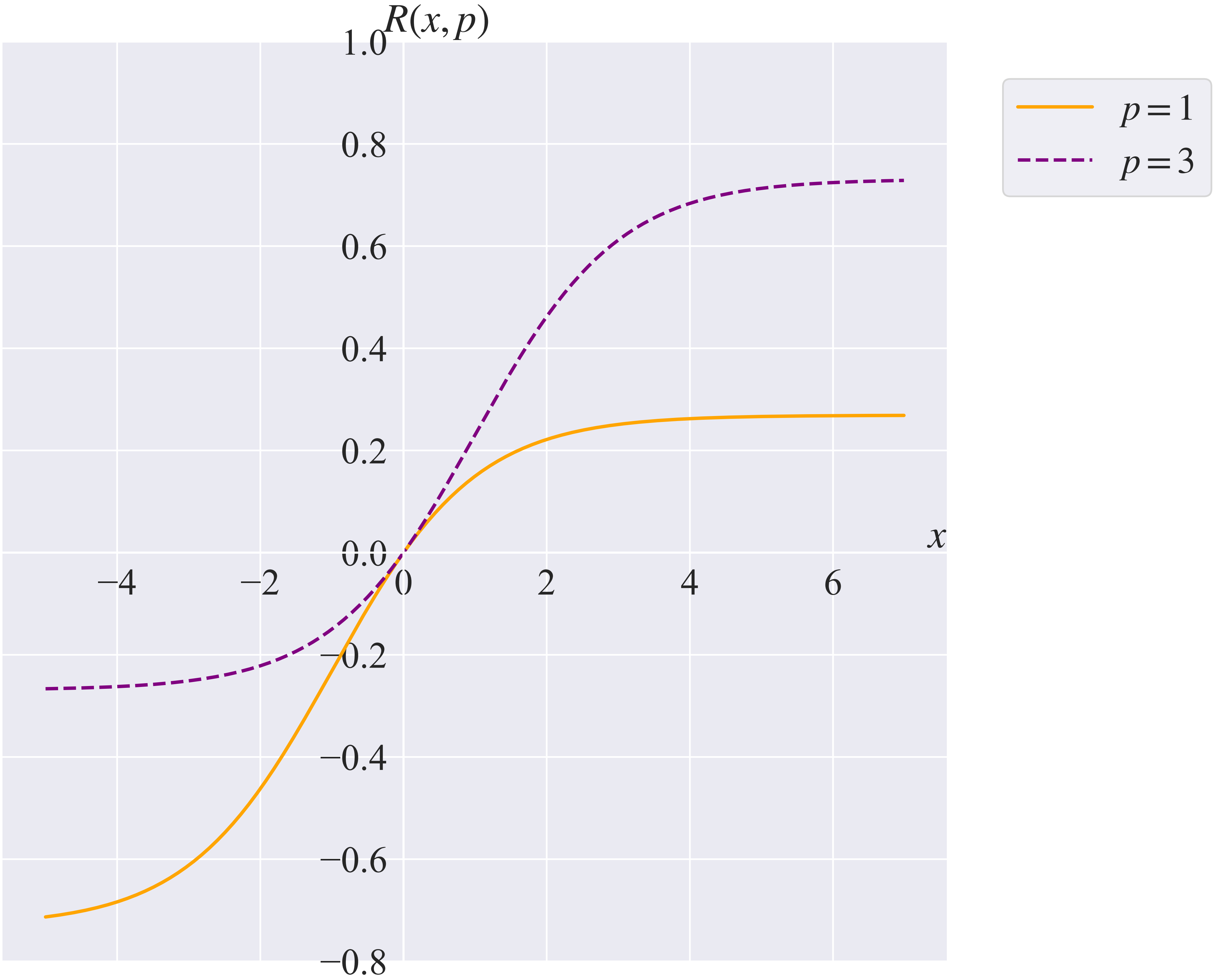

Figure 1: Dependence of reference effects on price

“Dimensioning Sensitivity” Property

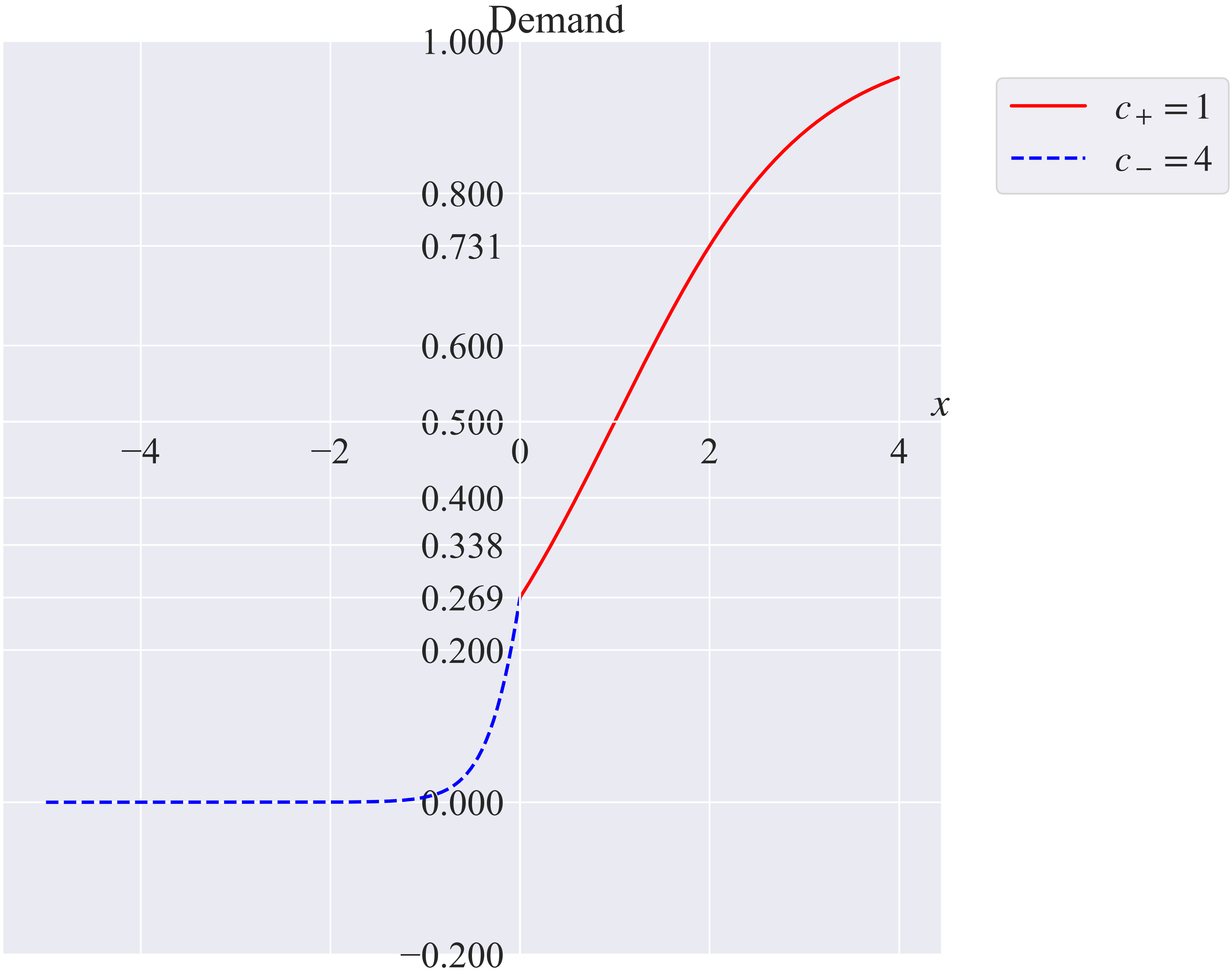

Figure 2: Examples of regional reference effects

Conditional Gradient Method

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Computing NPMLE via Conditional Gradient Method

Repeat

Find new consumer segment via solving subproblem

$$\mathbf{g}_k \cdot \nabla \ell(\mathbf{f}_k)$$

Re-maximize objective over new segment

$$\ell(\mathbf{f}), \text{ where } \mathbf{f} \in \operatorname{conv}(\mathbf{g}_1, \ldots, \mathbf{g}_{k-1})$$

$k \leftarrow k + 1$

Until convergence

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- Convergence rate can be established even when subproblem is solved only approximately
- New consumer segment is adaptively added to distribution

Myopic Pricing Policy

$$p_{\mathrm{m}}(r_t) = \arg\max_{p \in \mathscr{P}} \Pi(r_t, p)$$

- Likely sub-optimal but computationally efficient

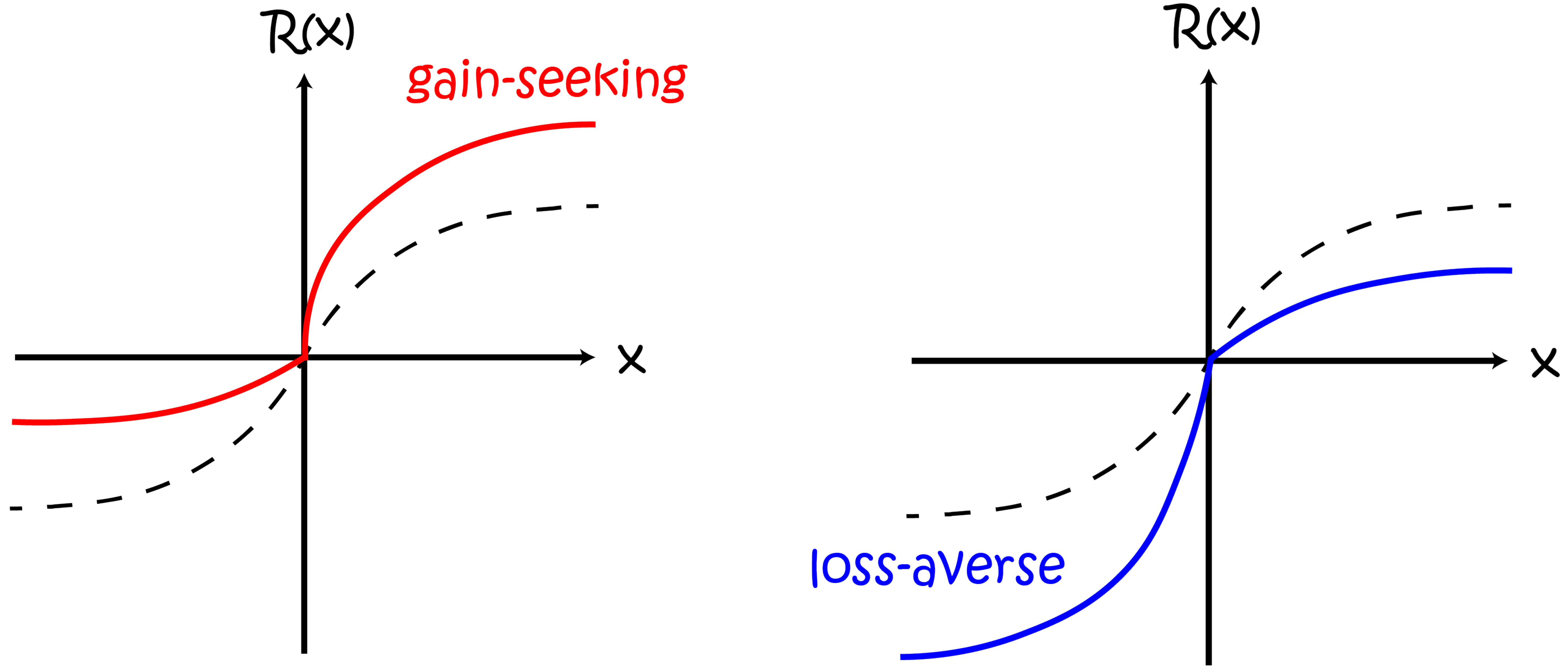
Proposition For any initial reference price r,

$$0 \leq V^*(r) - V_{\mathrm{m}}(r) \leq \frac{\beta(1-\alpha)}{(1-\alpha\beta)(1-\beta)} \eta(G) p_H$$

where $\eta(G) = \min\left(1, \sup_{(a,b,c_+,c_-) \in \mathrm{supp}(G)} \frac{\max(c_+, c_-)}{b + c_-}\right)$.

Reference Effects

- Reference discrepancy x: reference price r - current price p
- Reference effect $R(p)$: incurred demand change
- Frequent consumers perceive gains if $x > 0$ and losses if $x < 0$
- Consumers respond differently under reference effects



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When does the myopic pricing policy perform well?

- When memory parameter $\alpha \to 1$, reference prices are unchanged
- When discount factor $\beta \to 0$, less weights are allocated to future revenue